29. PROBABILITY

CONDITIONAL PROBABILITY AND PROBABILITY OF INDEPENDENT EVENTS

The concept of probability and various results on it were discussed in class XI. In this chapter, we shall be dealing with problems based on conditional probability and probability of independent events.

CONDITIONAL PROBABILITY

Let A and B be the two events associated with the same random experiment. Then, the probability of occurrence of A under the condition B has already occurred and $P(B) \neq 0$, is called conditional probability, denoted by $P(A/B)$.

We define:

$$P(A/B) = \frac{P(A \cap B)}{P(B)}, \text{ where } P(B) \neq 0$$

$$\text{and } P(B/A) = \frac{P(B \cap A)}{P(A)} = \frac{P(A \cap B)}{P(A)}, \text{ where } P(A) \neq 0.$$

SOLVED EXAMPLES

EXAMPLE 1 *A die is rolled. If the outcome is an odd number, what is the probability that it is prime?*

SOLUTION When a die is rolled, the sample space is given by

$$S = \{1, 2, 3, 4, 5, 6\}.$$

Let A = event of getting a prime number, and

B = event of getting an odd number.

Then, $A = \{2, 3, 5\}$, $B = \{1, 3, 5\}$ and $A \cap B = \{3, 5\}$.

$$\therefore \quad P(A) = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}, \; P(B) = \frac{n(B)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

$$\text{and } P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{2}{6} = \frac{1}{3}.$$

Suppose B has already occurred and then A occurs.

So, we have to find $P(A/B)$.

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Now, $P(A/B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{(1/3)}{(1/2)} = \left(\dfrac{1}{3} \times \dfrac{2}{1}\right) = \dfrac{2}{3}$.

$P(A/B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{(1/3)}{(1/2)} = \left(\dfrac{1}{3} \times \dfrac{2}{1}\right) = \dfrac{2}{3}$.

Hence, the required probability is $\dfrac{2}{3}$.

EXAMPLE 2 *Ten cards numbered* 1 *to* 10 *are placed in a box, mixed up thoroughly and then one card is drawn randomly. If it is known that the number on the drawn card is more than* 3, *what is the probability that it is an even number?*

SOLUTION Clearly, the sample space is $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

Let A = event that the number on the drawn card is even, and

B = event that the number on the drawn card is more than 3.

Then $A = \{2, 4, 6, 8, 10\}$, $B = \{4, 5, 6, 7, 8, 9, 10\}$

and $A \cap B = \{4, 6, 8, 10\}$.

$\therefore$ $P(A) = \dfrac{n(A)}{n(S)} = \dfrac{5}{10} = \dfrac{1}{2}$, $P(B) = \dfrac{n(B)}{n(S)} = \dfrac{7}{10}$ and

$P(A \cap B) = \dfrac{n(A \cap B)}{n(S)} = \dfrac{4}{10} = \dfrac{2}{5}$.

Suppose B has already occurred and then A occurs.

So, we have to find $P(A/B)$.

Now, $P(A/B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{(2/5)}{(7/10)} = \left(\dfrac{2}{5} \times \dfrac{10}{7}\right) = \dfrac{4}{7}$.

Hence, the required probability is $\dfrac{4}{7}$.

EXAMPLE 3 *A die is thrown twice and the sum of the numbers appearing is observed to be* 6. *What is the conditional probability that the number* 4 *has appeared at least once?*

SOLUTION We know that when a die is thrown twice, then the sample space has 36 possible outcomes.

Let A = event that 4 appears at least once, and

B = event that the sum of the numbers appearing is 6.

Then,

$A = \{(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (1, 4), (2, 4), (3, 4), (5, 4), (6, 4)\}$

and $B = \{(1, 5), (2, 4), (3, 3), (4, 2), (5, 1)\}$.

$\therefore$ $A \cap B = \{(2, 4), (4, 2)\}$.

So, $P(A) = \dfrac{n(A)}{n(S)} = \dfrac{11}{36}$, $P(B) = \dfrac{n(B)}{n(S)} = \dfrac{5}{36}$

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and $P(A \cap B) = \dfrac{n(A \cap B)}{n(S)} = \dfrac{2}{36} = \dfrac{1}{18}$.

Suppose B has already occurred and then A occurs.

So, we have to find $P(A/B)$.

Now, $P(A/B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{(1/18)}{(5/36)} = \left(\dfrac{1}{18} \times \dfrac{36}{5}\right) = \dfrac{2}{5}$.

Hence, the required probability is $\dfrac{2}{5}$.

EXAMPLE 4 *Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that (i) the youngest child is a girl, (ii) at least one of the children is a girl?*

SOLUTION We may write the sample space as

$S = \{G_1G_2, G_1B_2, B_1G_2, B_1B_2\}$, where the youngest child appears later.

(i) Let A = event that both the children are girls, and

B = event that the youngest child is a girl.

Then, $A = \{G_1G_2\}$, $B = \{G_1G_2, B_1G_2\}$ and $A \cap B = \{G_1G_2\}$.

$\therefore \quad P(A) = \dfrac{n(A)}{n(S)} = \dfrac{1}{4}, \quad P(B) = \dfrac{n(B)}{n(S)} = \dfrac{2}{4} = \dfrac{1}{2}$

and $P(A \cap B) = \dfrac{n(A \cap B)}{n(S)} = \dfrac{1}{4}$.

Suppose B has already occurred and then A occurs.

So, we have to find $P(A/B)$.

Now, $P(A/B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{(1/4)}{(1/2)} = \left(\dfrac{1}{4} \times \dfrac{2}{1}\right) = \dfrac{1}{2}$.

Hence, the required probability is $\dfrac{1}{2}$.

(ii) Let A = event that both the children are girls, and

E = event that at least one of the children is a girl.

Then, $A = \{G_1G_2\}$, $E = \{G_1B_2, B_1G_2, G_1G_2\}$ and $A \cap E = \{G_1G_2\}$.

$\therefore \quad P(A) = \dfrac{n(A)}{n(S)} = \dfrac{1}{4}, \quad P(E) = \dfrac{n(E)}{n(S)} = \dfrac{3}{4}$

and $P(A \cap E) = \dfrac{n(A \cap E)}{n(S)} = \dfrac{1}{4}$.

Suppose E has already occurred and then A occurs.

So, we have to find $P(A/E)$.

Now, $P(A/E) = \dfrac{P(A \cap E)}{P(E)} = \dfrac{(1/4)}{(3/4)} = \left(\dfrac{1}{4} \times \dfrac{4}{3}\right) = \dfrac{1}{3}$.

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Hence, the required probability is $\frac{1}{3}$.

EXAMPLE 5 *An instructor has a question bank consisting of* 300 *easy true/false questions;* 200 *difficult true/false questions;* 500 *easy multiple-choice questions and* 400 *difficult multiple-choice questions. If a question is selected at random from the question bank, what is the probability that it will be an easy question given that it is a multiple-choice question?*

SOLUTION Clearly, the sample space consists of 1400 questions.

$\therefore \quad n(S) = 1400.$

Let A = event of selecting an easy question, and

B = event of selecting a multiple-choice question.

Then, $A \cap B$ = event of selecting an easy multiple-choice question.

$\therefore \quad n(A) = (300 + 500) = 800, n(B) = (500 + 400) = 900$

and $n(A \cap B) = 500.$

So, $P(A) = \frac{n(A)}{n(S)} = \frac{800}{1400} = \frac{4}{7}, \quad P(B) = \frac{n(B)}{n(S)} = \frac{900}{1400} = \frac{9}{14}$

and $P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{500}{1400} = \frac{5}{14}.$

Suppose B has already occurred and then A occurs.

Thus we have to find $P(A/B)$.

Now, $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{(5/14)}{(9/14)} = \left(\frac{5}{14} \times \frac{14}{9}\right) = \frac{5}{9}.$

Hence, the required probability is $\frac{5}{9}$.

EXAMPLE 6 *Two numbers are selected at random from the integers* 1 *through* 9. *If the sum is even, find the probability that both the numbers are odd.*

SOLUTION Out of the numbers from 1 to 9, there are 5 odd numbers and 4 even numbers.

Let A = event of choosing two odd numbers, and

B = event of choosing two numbers whose sum is even.

Then, $n(A)$ = number of ways of choosing 2 odd numbers out of 5
$= {}^5C_2.$

$n(B)$ = number of ways of choosing 2 numbers whose sum is even
$= ({}^4C_2 + {}^5C_2)$ [2 out of 4 even and 2 out of 5 odd].

$n(A \cap B)$ = number of ways of choosing 2 odd numbers out of 5
$= {}^5C_2.$

Suppose B has already occurred and then A occurs.

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Then, we have to find $P(A/B)$.

Now, $P(A/B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{n(A \cap B)}{n(B)} = \dfrac{{}^5C_2}{{}^4C_2 + {}^5C_2}$

$= \dfrac{5 \times 4}{2} \times \dfrac{2}{(4 \times 3 + 5 \times 4)} = \dfrac{20}{32} = \dfrac{5}{8}.$

Hence, the required probability is $\dfrac{5}{8}$.

Properties of Conditional Probability

THEOREM 1 *Let A and B be the events of a sample space S of an experiment. Then, prove that $P(S/B) = P(B/B) = 1$.*

PROOF We know that:

$$P(S/B) = \frac{P(S \cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1 \ [\because \ S \cap B = B].$$

$$\text{And, } P(B/B) = \frac{P(B \cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1.$$

Hence, $P(S/B) = P(B/B) = 1$.

THEOREM 2 *Let A and B be the two events of a sample space S and let E be an event such that $P(E) \neq 0$. Then, prove that*

$$P[(A \cup B)/E] = P(A/E) + P(B/E) - P[(A \cap B)/E].$$

PROOF We have

$$P[(A \cup B)/E] = \frac{P[(A \cup B) \cap E]}{P(E)}$$

$$= \frac{P[(A \cap E) \cup (B \cap E)]}{P(E)}$$

$$= \frac{P(A \cap E) + P(B \cap E) - P[(A \cap E) \cap (B \cap E)]}{P(E)}$$

$$= \frac{P(A \cap E) + P(B \cap E) - P(A \cap B \cap E)}{P(E)}$$

$$= \frac{P(A \cap E)}{P(E)} + \frac{P(B \cap E)}{P(E)} - \frac{P[(A \cap B) \cap E]}{P(E)}$$

$$= P(A/E) + P(B/E) - P[(A \cap B)/E].$$

Hence, $P[(A \cup B)/E] = P(A/E) + P(B/E) - [(A \cap B)/E]$.

COROLLARY *If A and B are disjoint events, prove that*

$$P[(A \cup B)/E] = P(A/E) + P(B/E).$$

PROOF For any events A and B, we have

$$P[(A \cup B)/E] = P(A/E) + P(B/E) - P[(A \cap B)/E].$$

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If A and B are disjoint, then $P[(A \cap B)/E] = 0$.

Hence, in this case, we have

$$P[(A \cup B)/E] = P(A/E) + P(B/E).$$

THEOREM 3 *For any events A and B of a sample space S, prove that $P(\overline{A}/B) = 1 - P(A/B)$, where $\overline{A}$ denotes 'not A'.*

PROOF We know that

$$P(S/B) = 1 \Rightarrow P[(A \cup \overline{A})/B] = 1 \ [\because S = A \cup \overline{A}]$$
$$\Rightarrow P(A/B) + P(\overline{A}/B) = 1 \ [\because A \cap \overline{A} = \varphi]$$
$$\Rightarrow P(\overline{A}/B) = 1 - P(A/B).$$

Hence, $P(\overline{A}/B) = 1 - P(A/B)$.

EXAMPLE 7 *If A and B are the two events such that $P(A) = \frac{3}{5}$, $P(B) = \frac{7}{10}$ and $P(A \cup B) = \frac{9}{10}$, then find (i) $P(A \cap B)$ (ii) $P(A/B)$ (iii) $P(B/A)$.*

SOLUTION (i) We know that

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$
$$= \left(\frac{3}{5} + \frac{7}{10} - \frac{9}{10}\right) = \frac{(6+7-9)}{10} = \frac{4}{10} = \frac{2}{5}.$$

(ii) $$P(A/B) = \frac{(A \cap B)}{P(B)} = \frac{(2/5)}{(7/10)} = \left(\frac{2}{5} \times \frac{10}{7}\right) = \frac{4}{7}.$$

(iii) $$P(B/A) = \frac{(B \cap A)}{P(A)} = \frac{P(A \cap B)}{P(A)} = \left(\frac{2/5}{3/5}\right) = \left(\frac{2}{5} \times \frac{5}{3}\right) = \frac{2}{3}.$$

EXAMPLE 8 *Evaluate $P(A \cup B)$, if $2P(A) = P(B) = \frac{6}{13}$ and $P(A/B) = \frac{1}{3}$.*

SOLUTION $$2P(A) = P(B) = \frac{6}{13} \Rightarrow P(A) = \frac{3}{13} \text{ and } P(B) = \frac{6}{13}.$$

$$\therefore \quad P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow P(A \cap B) = P(A/B) \cdot P(B) = \left(\frac{1}{3} \times \frac{6}{13}\right) = \frac{2}{13}.$$

So, $$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= \left(\frac{3}{13} + \frac{6}{13} - \frac{2}{13}\right) = \frac{(3+6-2)}{13} = \frac{7}{13}.$$

Hence, $P(A \cup B) = \frac{7}{13}$.

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EXAMPLE 9 *Let A and B be the events such that* $P(A)=\frac{1}{3}$, $P(B)=\frac{1}{4}$ *and* $P(A\cap B)=\frac{1}{5}$.

Find: (i) $P(A/B)$ *(ii)* $P(B/A)$ *(iii)* $P(A\cup B)$ *(iv)* $P(\overline{B}/\overline{A})$

SOLUTION We have:

$$\text{(i) } P(A/B)=\frac{P(A\cap B)}{P(B)}=\frac{(1/5)}{(1/4)}=\left(\frac{1}{5}\times\frac{4}{1}\right)=\frac{4}{5}.$$

$$\text{(ii) } P(B/A)=\frac{P(B\cap A)}{P(A)}=\frac{P(A\cap B)}{P(A)}=\frac{(1/5)}{(1/3)}=\left(\frac{1}{5}\times\frac{3}{1}\right)=\frac{3}{5}.$$

$$\text{(iii) } P(A\cup B)=P(A)+P(B)-P(A\cap B)$$
$$=\left(\frac{1}{3}+\frac{1}{4}-\frac{1}{5}\right)=\frac{(20+15-12)}{60}=\frac{23}{60}.$$

$$\text{(iv) } P(\overline{B}/\overline{A})=\frac{P(\overline{B}\cap\overline{A})}{P(\overline{A})}=\frac{P(\overline{A}\cap\overline{B})}{P(\overline{A})}=\frac{P(\overline{A\cup B})}{P(\overline{A})}$$
$$=\frac{1-P(A\cup B)}{1-P(A)}=\frac{\left(1-\frac{23}{60}\right)}{\left(1-\frac{1}{3}\right)}=\frac{\left(\frac{37}{60}\right)}{\left(\frac{2}{3}\right)}=\left(\frac{37}{60}\times\frac{3}{2}\right)=\frac{37}{40}.$$

EXERCISE 29A

1. Let A and B be the events such that
$P(A)=\frac{7}{13}$, $P(B)=\frac{9}{13}$ and $P(A\cap B)=\frac{4}{13}$.
Find (i) $P(A/B)$ (ii) $P(B/A)$ (iii) $P(A\cup B)$ (iv) $P(\overline{B}/\overline{A})$.

2. Let A and B be the events such that
$P(A)=\frac{5}{11}$, $P(B)=\frac{6}{11}$ and $P(A\cup B)=\frac{7}{11}$.
Find (i) $P(A\cap B)$ (ii) $P(A/B)$ (iii) $P(B/A)$ (iv) $P(\overline{A}/\overline{B})$.

3. Let A and B be the events such that
$P(A)=\frac{3}{10}$, $P(B)=\frac{1}{2}$ and $P(B/A)=\frac{2}{5}$.
Find (i) $P(A\cap B)$ (ii) $P(A\cup B)$ (iii) $P(A/B)$.

4. Let A and B be the events such that
$2P(A)=P(B)=\frac{5}{13}$ and $P(A/B)=\frac{2}{5}$.
Find (i) $P(A\cap B)$ (ii) $P(A\cup B)$.

5. A die is rolled. If the outcome is an even number, what is the probability that it is a number greater than 2?

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6. A coin is tossed twice. If the outcome is at most one tail, what is the probability that both head and tail have appeared?
7. Three coins are tossed simultaneously. Find the probability that all coins show heads if at least one of the coins shows a head.
8. Two unbiased dice are thrown. Find the probability that the sum of the numbers appearing is 8 or greater, if 4 appears on the first die.
9. A die is thrown twice and the sum of the numbers appearing is observed to be 8. What is the conditional probability that the number 5 has appeared at least once?
10. Two dice were thrown and it is known that the numbers which come up were different. Find the probability that the sum of the two numbers was 5.
11. A coin is tossed and then a die is thrown. Find the probability of obtaining a 6, given that a head came up.
12. A couple has 2 children. Find the probability that both are boys if it is known that (i) one of the children is a boy, and (ii) the elder child is a boy.
13. In a class, 40% students study mathematics; 25% study biology and 15% study both mathematics and biology. One student is selected at random. Find the probability that
    (i) he studies mathematics if it is known that he studies biology
    (ii) he studies biology if it is known that he studies mathematics.
14. The probability that a student selected at random from a class will pass in Hindi is $\frac{4}{5}$ and the probability that he passes in Hindi and English is $\frac{1}{2}$. What is the probability that he will pass in English if it is known that he has passed in Hindi?
15. The probability that a certain person will buy a shirt is 0.2, the probability that he will buy a coat is 0.3 and the probability that he will buy a shirt given that he buys a coat is 0.4. Find the probability that he will buy both a shirt and a coat.
16. In a hostel, 60% of the students read Hindi newspaper, 40% read English newspaper and 20% read both Hindi and English newspapers. A student is selected at random.
    (i) Find the probability that he reads neither Hindi nor English newspaper.
    (ii) If he reads Hindi newspaper, what is the probability that he reads English newspaper?
    (iii) If he reads English newspaper, what is the probability that he reads Hindi newspaper?
17. Two integers are selected at random from integers 1 through 11. If the sum is even, find the probability that both the numbers selected are odd.

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ANSWERS (EXERCISE 29A)

1. (i) $\frac{4}{9}$ (ii) $\frac{4}{7}$ (iii) $\frac{12}{13}$ (iv) $\frac{1}{6}$ **2.** (i) $\frac{4}{11}$ (ii) $\frac{2}{3}$ (iii) $\frac{4}{5}$ (iv) $\frac{4}{5}$

3. (i) $\frac{3}{25}$ (ii) $\frac{17}{25}$ (iii) $\frac{6}{25}$ **4.** (i) $\frac{2}{13}$ (ii) $\frac{11}{26}$ **5.** $\frac{2}{3}$ **6.** $\frac{2}{3}$ **7.** $\frac{1}{7}$

8. $\frac{1}{2}$ **9.** $\frac{2}{5}$ **10.** $\frac{2}{15}$ **11.** $\frac{1}{6}$ **12.** (i) $\frac{1}{3}$ (ii) $\frac{1}{2}$ **13.** (i) $\frac{3}{5}$ (ii) $\frac{3}{8}$

14. $\frac{5}{8}$ **15.** 0.12 **16.** (i) $\frac{1}{5}$ (ii) $\frac{1}{3}$ (iii) $\frac{1}{2}$ **17.** $\frac{3}{5}$

HINTS TO SOME SELECTED QUESTIONS (EXERCISE 29A)

8. Let A = event of getting the sum 8 or greater, and
B = event of getting a 4 on the first die.

$\therefore$ $A = \{(2,6),(3,5),(3,6),(4,4),(4,5),(4,6),(5,3),(5,4),(5,5),(5,6),$
$(6,2),(6,3),(6,4),(6,5),(6,6)\}$

$B = \{(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)\}$.

$(A \cap B) = \{(4,4),(4,5),(4,6)\}$.

$\therefore$ $n(A) = 15, n(B) = 6$ and $n(A \cap B) = 3$.

Hence, the required probability $= P(A/B) = \frac{n(A \cap B)}{n(B)} = \frac{3}{6} = \frac{1}{2}$.

10. Let A = event that the sum of two numbers appeared is 5, and
B = event that the two dice show different numbers.

Then, $A = \{(2,3),(3,2),(1,4),(4,1)\}$.

Thus, $n(A) = 4, n(B) = 30, A \cap B = A$ and so $n(A \cap B) = n(A) = 4$.

$\therefore$ the required probability $= P(A/B) = \frac{n(A \cap B)}{n(B)} = \frac{4}{30} = \frac{2}{15}$.

11. Let A = event that the die shows a 6, and
B = event that a head comes up.

Then, $A = \{(H,6),(T,6)\}$

and $B = \{(H,1),(H,2),(H,3),(H,4),(H,5),(H,6)\}$.

Then, $P(A/B) = \frac{n(A \cap B)}{n(B)} = \frac{1}{6}$.

13. Let M = event of studying mathematics, and
B = event of studying biology.

Then $P(M) = \frac{40}{100} = \frac{2}{5}$, $P(B) = \frac{25}{100} = \frac{1}{4}$ and $P(M \cap B) = \frac{15}{100} = \frac{3}{20}$.

(i) $P(M/B) = \frac{P(M \cap B)}{P(B)}$.

(ii) $P(B/M) = \frac{P(M \cap B)}{P(M)}$.

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14. Given, $P(H)=\frac{4}{5}$ and $P(H\cap E)=\frac{1}{2}$.

$\therefore\quad P(E/H)=\frac{P(E\cap H)}{P(H)}=\frac{P(H\cap E)}{P(H)}$.

15. $P(S)=0.2, P(C)=0.3, P(S/C)=0.4$.

$\therefore\quad P(S/C)=\frac{P(S\cap C)}{P(C)}\Rightarrow\ P(S\cap C)=P(C)\cdot P(S/C)$.

17. Let $A=$ event of choosing both odd numbers,

$B=$ event that sum of chosen numbers is even.

In integers from 1 to 11, there are 5 even and 6 odd integers.

$$P(A)=\frac{{}^6C_2}{{}^{11}C_2}=\frac{3}{11},\ P(B)=\left(\frac{{}^6C_2+{}^5C_2}{{}^{11}C_2}\right)=\frac{5}{11},\ P(A\cap B)=\frac{{}^6C_2}{{}^{11}C_2}=\frac{3}{11}.$$

$\therefore$ required probability $=P(A/B)=\frac{P(A\cap B)}{P(B)}$.

Multiplication Theorem on Probability

Let A and B be the two events associated with a sample space S. Then, the simultaneous occurrence of two events A and B is denoted by $(A\cap B)$ and also written as AB.

MULTIPLICATION THEOREM *Let A and B be the two events associated with a sample space S. Then, prove that*

$$P(AB)=P(A\cap B)=P(A)\cdot P(B/A)=P(B)\cdot P(A/B),$$

provided $P(A)\neq 0$ and $P(B)\neq 0$.

PROOF For any events A and B, we have

$$P(A/B)=\frac{P(A\cap B)}{P(B)},\text{ where } P(A)\neq 0.$$

$\therefore\quad P(A\cap B)=P(B)\cdot P(A/B)$. ... (i)

Again, $P(B/A)=\frac{P(B\cap A)}{P(A)}=\frac{P(A\cap B)}{P(A)}$, where $P(B)\neq 0$.

$\therefore\quad P(A\cap B)=P(A)\cdot P(B/A)$. ... (ii)

From (i) and (ii), we get

$$P(A\cap B)=P(B)\cdot P(A/B)=P(A)\cdot P(A/B),\text{ where } P(A)\neq 0.$$

Multiplication Rule for Three Events

For any three events A, B, C of the same sample space, we have

$$P(A\cap B\cap C)=P(A)\cdot P(B/A)\cdot P[C/(A\cap B)]$$
$$=P(A)\cdot P(B/A)\cdot(C/AB).$$

This rule can be extended for four or more events.

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SOLVED EXAMPLES

EXAMPLE 1 *An urn contains 8 white and 4 red balls. Two balls are drawn from the urn one after the other without replacement. What is the probability that both drawn balls are white?*

SOLUTION Let A and B denote respectively the events that first and second balls both drawn are white.

Then, we have to find $P(A \cap B)$.

Now, $P(A) = P$ (white ball in the first draw) $= \frac{8}{12}$.

After the occurrence of event A, we are left with 7 white and 4 red balls.

The probability of drawing second white ball, given that the first ball drawn is white, is clearly the conditional probability of occurrence of B, given that A has occurred.

$\therefore \quad P(B/A) = \frac{7}{11}$.

By multiplication rule of probability, we have

$$P(A \cap B) = P(A) \cdot P(B/A) = \left(\frac{8}{12} \times \frac{7}{11}\right) = \frac{14}{33}.$$

Hence, the required probability is $\frac{14}{33}$.

EXAMPLE 2 *Three cards are drawn successively without replacement from a pack of 52 well-shuffled cards. What is the probability that first two cards are queens and the third card drawn is a king?*

SOLUTION Let Q denote the event that the card drawn is a queen and K be the event that the card drawn is a king. Then, we have to find $P(QQK)$.

Probability of drawing first queen is $P(Q) = \frac{4}{52}$.

Now, there are 3 queens in remaining 51 cards.

Let $P(Q/Q)$ be the probability of getting the second queen with the condition that one queen has already been drawn.

$\therefore \quad P(Q/Q) = \frac{3}{51}$.

Lastly, $P(K/QQ)$ is probability of third drawn card to be a king, with the condition that two queens have already been drawn.

Now, there are 4 kings in remaining 50 cards.

$\therefore \quad P(K/QQ) = \frac{4}{50}$.

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By multiplication law of probability, we have

$$P(QQK) = P(Q \cap Q \cap K)$$
$$= P(Q) \cdot P(Q/Q) \cdot P(K/QQ)$$
$$= \left(\frac{4}{52} \times \frac{3}{51} \times \frac{4}{50}\right) = \left(\frac{1}{13} \times \frac{1}{17} \times \frac{2}{25}\right) = \frac{2}{5525}.$$

Hence, the required probability is $\frac{2}{5525}$.

Independent Events

Two events A and B are said to be independent if

$P(A/B) = P(A)$, where $P(B) \neq 0$

and $P(B/A) = P(B)$, where $P(A) \neq 0$.

i.e., the event A does not depend on the occurence of event B and vice versa.

Condition for Independence of Two Events

For any two events A and B, we have

$P(A \cap B) = P(A) \cdot P(B/A)$. ... (i)

If A and B are independent, we have $P(B/A) = P(B)$.

$\therefore$ (i) becomes $\boldsymbol{P(A \cap B) = P(A) \times P(B).}$

Thus, *two events A and B associated with the same random experiment are said to be independent if* $P(A \cap B) = P(A) \times P(B)$.

NOTE: Two events A and B are said to be dependent if they are not independent, i.e., if $P(A \cap B) \neq P(A) \times P(B)$.

Difference between Two Mutually Exclusive and Independent Events

Two events A and B are said to be mutually exclusive if $A \cap B = \varphi$ and in this case $P(A \cap B) = P(\varphi) = 0$.

Also we know that two events A and B are independent if $P(A \cap B) = P(A) \times P(B)$.

Clearly, two independent events with nonzero probabilities cannot be mutually exclusive.

Also, two mutually exclusive events with nonzero probabilities cannot be mutually independent.

Three events A, B and C are said to be mutually independent, if

$P(A \cap B) = P(A) \times P(B)$, $P(A \cap C) = P(A) \times P(C)$, $P(B \cap C) = P(B) \times P(C)$

and $P(A \cap B \cap C) = P(A) \times P(B) \times P(C)$.

If at least one of the above is not true for three given events A, B and C, then we say that these events are not independent.

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EXAMPLE 3 *Let E_1 and E_2 be two events such that $P(E_1) = 0.3$, $P(E_1 \cup E_2) = 0.4$ and $P(E_2) = x$. Find the value of x such that*
(i) E_1 and E_2 are mutually exclusive,
(ii) E_1 and E_2 are independent.

SOLUTION (i) Let E_1 and E_2 be mutually exclusive. Then, $E_1 \cap E_2 = \varphi$.

$\therefore \quad P(E_1 \cup E_2) = P(E_1) + P(E_2)$

$\Rightarrow \quad 0.4 = 0.3 + x$

$\Rightarrow \quad x = 0.1.$

Thus, when E_1 and E_2 mutually exclusive, then $x = 0.1$.

(ii) Let E_1 and E_2 be two independent events. Then,

$$P(E_1 \cap E_2) = P(E_1) \times P(E_2) = 0.3 \times x = 0.3x.$$

$\therefore \quad P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$\Rightarrow \quad 0.4 = 0.3 + x - 0.3x$

$\Rightarrow \quad 0.7x = 0.1$

$\Rightarrow \quad x = \dfrac{0.1}{0.7} = \dfrac{1}{7}.$

Thus, when E_1 and E_2 are independent, then $x = \dfrac{1}{7}$.

EXAMPLE 4 *Let E_1 and E_2 are the two independent events such that $P(E_1) = 0.35$ and $P(E_1 \cup E_2) = 0.60$, find $P(E_2)$.*

SOLUTION Let $P(E_2) = x$.

Then, E_1 and E_2 being independent events, we have

$$P(E_1 \cap E_2) = P(E_1) \times P(E_2) = 0.35 \times x = 0.35x.$$

Now, $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$\Rightarrow \quad 0.60 = 0.35 + x - 0.35x$

$\Rightarrow \quad 0.65x = 0.25$

$\Rightarrow \quad x = \dfrac{0.25}{0.65} = \dfrac{25}{65} = \dfrac{5}{13}.$

Hence, $P(E_2) = \dfrac{5}{13}$.

EXAMPLE 5 *A coin is tossed thrice. Let the event E be 'the first throw results in a head', and the event F be 'the last throw results in a tail'. Find whether the events E and F are independent.*

SOLUTION When a coin is tossed three times, the sample space is given by

$$S = \{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT\}.$$

Now, E = event that the first throw results in a head.

$\therefore \quad E = \{HHH, HHT, HTH, HTT\}.$

And, F = event that the last throw results in a tail.

$\therefore \quad F = \{HHT, THT, HTT, TTT\}.$

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So, $(E \cap F) = \{HHT, HTT\}$.

Clearly, $n(E) = 4$, $n(F) = 4$, $n(E \cap F) = 2$ and $n(S) = 8$.

$\therefore \quad P(E) = \dfrac{n(E)}{n(S)} = \dfrac{4}{8} = \dfrac{1}{2}$, $P(F) = \dfrac{n(F)}{n(S)} = \dfrac{4}{8} = \dfrac{1}{2}$

and $P(E \cap F) = \dfrac{n(E \cap F)}{n(S)} = \dfrac{2}{8} = \dfrac{1}{4}$.

Thus, $P(E \cap F) = P(E) \times P(F)$.

Hence, E and F are independent events.

EXAMPLE 6 *An unbiased die is tossed twice. Find the probability of getting a* 4, 5 *or* 6 *on the first toss and a* 1, 2, 3 *or* 4 *on the second toss.*

SOLUTION In each case, the sample space is given by $S = \{1, 2, 3, 4, 5, 6\}$.

Let E = event of getting a 4, 5 or 6 on the first toss.

And, F = event of getting a 1, 2, 3 or 4 on the second toss.

Then, $P(E) = \dfrac{3}{6} = \dfrac{1}{2}$ and $P(F) = \dfrac{4}{6} = \dfrac{2}{3}$.

Clearly, E and F are independent events.

$\therefore$ required probability $= P(E \cap F) = P(E) \times P(F)$

[$\because$ E and F are independent]

$$= \left(\frac{1}{2} \times \frac{2}{3}\right) = \frac{1}{3}.$$

EXAMPLE 7 *Ramesh appears for an interview for two posts, A and B, for which the selection is independent. The probability for his selection for Post A is* $(1/6)$ *and for Post B, it is* $(1/7)$. *Find the probability that Ramesh is selected for at least one post.* **[CBSE 2001]**

SOLUTION Let E_1 = event that Ramesh is selected for the post A,

and E_2 = event that Ramesh is selected for the post B.

Then, $P(E_1) = \dfrac{1}{6}$ and $P(E_2) = \dfrac{1}{7}$.

Clearly, E_1 and E_2 are independent events.

$\therefore \quad P(E_1 \cap E_2) = P(E_1) \times P(E_2) = \left(\dfrac{1}{6} \times \dfrac{1}{7}\right) = \dfrac{1}{42}$.

$\therefore$ P(Ramesh is selected for at least one post)

$$= P(E_1 \cup E_2)$$
$$= P(E_1) + P(E_2) - P(E_1 \cap E_2)$$
$$= \left(\frac{1}{6} + \frac{1}{7} - \frac{1}{42}\right) = \frac{12}{42} = \frac{2}{7}.$$

Hence, the required probability is $\dfrac{2}{7}$.

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EXAMPLE 8 A *can solve* 90% *of the problems given in a book, and* B *can solve* 70%. *What is the probability that at least one of them will solve a problem selected at random from the book?*

SOLUTION Let E_1 = event that A solves the problem,
and E_2 = event that B solves the problem.

Then, $P(E_1) = \frac{90}{100} = \frac{9}{10}$ and $P(E_2) = \frac{70}{100} = \frac{7}{10}$.

Clearly, E_1 and E_2 are independent events.

$$\therefore \quad P(E_1 \cap E_2) = P(E_1) \times P(E_2) = \left(\frac{9}{10} \times \frac{7}{10}\right) = \frac{63}{100}.$$

$\therefore$ P(at least one of them will solve the problem)

$$= P(E_1 \cup E_2)$$
$$= P(E_1) + P(E_2) - P(E_1 \cap E_2)$$
$$= \left(\frac{9}{10} + \frac{7}{10} - \frac{63}{100}\right) = \frac{(90+70-63)}{100} = \frac{97}{100}.$$

Hence, the required probability is 0.97.

EXAMPLE 9 *The probability that* A *hits a target is* (1/3) *and the probability that B hits it is* (2/5). *What is the probability that the target will be hit if both A and* B *shoot at it?*

SOLUTION Let E_1 = event that A hits the target,
and E_2 = event that B hits the target.

Then, $P(E_1) = \frac{1}{3}$ and $P(E_2) = \frac{2}{5}$.

Clearly, E_1 and E_2 are independent events.

$$\therefore \quad P(E_1 \cap E_2) = P(E_1) \times P(E_2) = \left(\frac{1}{3} \times \frac{2}{5}\right) = \frac{2}{15}.$$

$\therefore$ P(target is hit) = P(A hits or B hits)

$$= P(E_1 \cup E_2)$$
$$= P(E_1) + P(E_2) - P(E_1 \cap E_2)$$
$$= \left(\frac{1}{3} + \frac{2}{5} - \frac{2}{15}\right) = \frac{9}{15} = \frac{3}{5}.$$

Hence, the required probability is $\frac{3}{5}$.

EXAMPLE 10 A *and* B *appear for an interview for two posts. The probability of* A*'s selection is* (1/3) *and that of* B*'s selection is* (2/5). *Find the probability that only one of them will be selected.*

SOLUTION Let E_1 = event that A is selected,
and E_2 = event that B is selected.

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Then, $P(E_1) = \frac{1}{3}$ and $P(E_2) = \frac{2}{5}$

$\Rightarrow \quad P(\overline{E}_1) = \left(1 - \frac{1}{3}\right) = \frac{2}{3}$ and $P(\overline{E}_2) = \left(1 - \frac{2}{5}\right) = \frac{3}{5}$.

$\therefore$ P(event that only one of them is selected)

$= P[(E_1$ and not $E_2)$ or $(E_2$ and not $E_1)]$

$= P[(E_1 \cap \overline{E}_2)$ or $(E_2 \cap \overline{E}_1)]$

$= P(E_1 \cap \overline{E}_2) + P(E_2 \cap \overline{E}_1) \qquad [\because \ (E_1 \cap \overline{E}_2) \cap (E_2 \cap \overline{E}_1) = \phi]$

$= P(E_1) \cdot P(\overline{E}_2) + P(E_2) \cdot P(\overline{E}_1)$

$\left[\begin{array}{l} \because\ E_1 \text{ and } \overline{E}_2 \text{ are independent,} \\ \text{and } E_2 \text{ and } \overline{E}_1 \text{ are independent} \end{array}\right]$

$= \left(\frac{1}{3} \times \frac{3}{5}\right) + \left(\frac{2}{5} \times \frac{2}{3}\right) = \left(\frac{1}{5} + \frac{4}{15}\right) = \frac{7}{15}$.

EXAMPLE 11 *A speaks the truth in 60% of the cases, and* B *in 90% of the cases. In what percentage of cases are they likely to contradict each other in stating the same fact?* **[CBSE 2001]**

SOLUTION Let E_1 = event that A speaks the truth,

and E_2 = event that B speaks the truth.

Then, $\overline{E}_1$ = event that A tells a lie,

and $\overline{E}_2$ = event that B tells a lie.

Clearly, E_1 and E_2 are independent events.

Also, $(E_1$ and $\overline{E}_2)$ as well as $(\overline{E}_1$ and $E_2)$ are independent.

Now, $P(E_1) = \frac{60}{100} = \frac{3}{5}$; $P(E_2) = \frac{90}{100} = \frac{9}{10}$;

$$P(\overline{E}_1) = \left(1 - \frac{3}{5}\right) = \frac{2}{5} \text{ and } P(\overline{E}_2) = \left(1 - \frac{9}{10}\right) = \frac{1}{10}.$$

$\therefore$ P(A and B contradict each other)

$= P[$(A speaks the truth and B tells a lie)

or (A tells a lie and B speaks the truth)$]$

$= P[(E_1 \cap \overline{E}_2) \cup (\overline{E}_1 \cap E_2)]$

$= P(E_1 \cap \overline{E}_2) + P(\overline{E}_1 \cap E_2) \ [\because (E_1 \cap \overline{E}_2) \cap (\overline{E}_1 \cap E_2) = \phi]$

$= \{P(E_1) \times P(\overline{E}_2)\} + \{P(\overline{E}_1) \times P(E_2)\}$

$= \left(\frac{3}{5} \times \frac{1}{10}\right) + \left(\frac{2}{5} \times \frac{9}{10}\right) = \left(\frac{3}{50} + \frac{18}{50}\right) = \frac{21}{50}$.

Percentage of cases in which A and B contradict each other

$= \left(\frac{21}{50} \times 100\right)\% = 42\%$.

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EXAMPLE 12 *The probabilities of a specific problem being solved independently by* A *and* B *are* ½ *and* ⅓ *respectively. If both try to solve the problem independently, find the probability that*

(i) the problem is solved

(ii) exactly one of them solves the problem.

SOLUTION Let E_1 = event that A solves the problem,

and E_2 = event that B solves the problem.

Then, $P(E_1) = \frac{1}{2}$ and $P(E_2) = \frac{1}{3}$

$\Rightarrow \; P(\overline{E}_1) = \left(1 - \frac{1}{2}\right) = \frac{1}{2}$ and $P(\overline{E}_2) = \left(1 - \frac{1}{3}\right) = \frac{2}{3}$.

Clearly, E_1 and E_2 are independent events.

$\therefore \; P(E_1 \cap E_2) = P(E_1) \times P(E_2) = \left(\frac{1}{2} \times \frac{1}{3}\right) = \frac{1}{6}$.

(i) P(the problem is solved)

$= P$(at least one of A and B solves the problem)

$= P(E_1 \text{ or } E_2) = P(E_1 \cup E_2)$

$= P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$= \left(\frac{1}{2} + \frac{1}{3} - \frac{1}{6}\right) = \frac{4}{6} = \frac{2}{3}$.

(ii) P(exactly one of them solves the problem)

$= P[(E_1 \text{ and not } E_2) \text{ or } (E_2 \text{ and not } E_1)]$

$= P(E_1 \text{ and not } E_2) + P(E_2 \text{ and not } E_1)$

$= P(E_1 \cap \overline{E}_2) + P(E_2 \cap \overline{E}_1)$

$= P(E_1) \times P(\overline{E}_2) + P(E_2) \times P(\overline{E}_1)$

$= \left(\frac{1}{2} \times \frac{2}{3}\right) + \left(\frac{1}{3} \times \frac{1}{2}\right) = \left(\frac{1}{3} + \frac{1}{6}\right) = \frac{3}{6} = \frac{1}{2}$.

EXAMPLE 13 *Amit and Nisha appear for an interview for two vacancies in a company. The probability of Amit's selection is* ⅕ *and that of Nisha's selection is* ⅙. *What is the probability that*

(i) both of them are selected?

(ii) only one of them is selected?

(iii) none of them is selected?

SOLUTION Let E_1 = event that Amit is selected,

and E_2 = event that Nisha is selected.

Then, $P(E_1) = \frac{1}{5}$ and $P(E_2) = \frac{1}{6}$.

Clearly, E_1 and E_2 are independent events.

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(i) P(both are selected) $= P(E_1 \cap E_2)$

$= P(E_1) \times P(E_2)$

[$\because$ E_1 and E_2 are independent]

$= \left(\frac{1}{5} \times \frac{1}{6}\right) = \frac{1}{30}$.

(ii) P(only one of them is selected)

$= P[(E_1 \text{ and not } E_2) \text{ or } (E_2 \text{ and not } E_1)]$

$= P(E_1 \text{ and not } E_2) + P(E_2 \text{ and not } E_1)$

$= P(E_1) \cdot P(\text{not } E_2) + P(E_2) \cdot P(\text{not } E_1)$

$= P(E_1) \cdot [1 - P(E_2)] + P(E_2) \cdot [1 - P(E_1)]$

$= \frac{1}{5} \cdot \left(1 - \frac{1}{6}\right) + \frac{1}{6} \cdot \left(1 - \frac{1}{5}\right) = \left(\frac{1}{5} \times \frac{5}{6}\right) + \left(\frac{1}{6} \times \frac{4}{5}\right)$

$= \left(\frac{1}{6} + \frac{2}{15}\right) = \frac{9}{30} = \frac{3}{10}$.

(iii) P (none of them is selected)

$= P(\text{not } E_1 \text{ and not } E_2)$

$= P(\text{not } E_1) \text{ and } P(\text{not } E_2)$

$= [1 - P(E_1)] \cdot [1 - P(E_2)]$

$= \left(1 - \frac{1}{5}\right) \cdot \left(1 - \frac{1}{6}\right) = \left(\frac{4}{5} \times \frac{5}{6}\right) = \frac{2}{3}$.

EXAMPLE 14 *Three groups of children contain* 3 *girls and* 1 *boy;* 2 *girls and* 2 *boys; and* 1 *girl and* 3 *boys. One child is selected at random from each group. Find the chance that the three children selected comprise* 1 *girl and* 2 *boys.*

SOLUTION Let G_1, G_2, G_3 be the events of selecting a girl from the first, second and third group respectively, and let B_1, B_2, B_3 be the events of selecting a boy from the first, second and third group respectively. Then,

$P(G_1) = \frac{3}{4}; P(G_2) = \frac{2}{4} = \frac{1}{2}; P(G_3) = \frac{1}{4}$.

$P(B_1) = \frac{1}{4}; P(B_2) = \frac{2}{4} = \frac{1}{2}$ and $P(B_3) = \frac{3}{4}$.

$\therefore$ P(selecting 1 girl and 2 boys)

$= P[(G_1B_2B_3) \text{ or } (B_1G_2B_3) \text{ or } (B_1B_2G_3)]$

$= P(G_1B_2B_3) + P(B_1G_2B_3) + P(B_1B_2G_3)$

$= \{P(G_1) \times P(B_2) \times P(B_3)\} + \{P(B_1) \times P(G_2) \times P(B_3)\}$

$+ \{P(B_1) \times P(B_2) \times P(G_3)\}$

$= \left(\frac{3}{4} \times \frac{1}{2} \times \frac{3}{4}\right) + \left(\frac{1}{4} \times \frac{1}{2} \times \frac{3}{4}\right) + \left(\frac{1}{4} \times \frac{1}{2} \times \frac{1}{4}\right) = \left(\frac{9}{32} + \frac{3}{32} + \frac{1}{32}\right) = \frac{13}{32}$.

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Hence, the chances of selecting 1 girl and 2 boys are $\frac{13}{32}$.

EXAMPLE 15 *A problem is given to three students whose chances of solving it are $\frac{1}{3}$, $\frac{2}{7}$ and $\frac{3}{8}$. What is the probability that the problem will be solved?*

SOLUTION Let the three students be named A, B, and C respectively. Let E_1, E_2, E_3 be the events that the problem is solved by A, B, C respectively. Then,

$$P(E_1) = \frac{1}{3},\ P(E_2) = \frac{2}{7},\ P(E_3) = \frac{3}{8}.$$

$$\therefore \quad P(\overline{E}_1) = \left(1 - \frac{1}{3}\right) = \frac{2}{3};\ P(\overline{E}_2) = \left(1 - \frac{2}{7}\right) = \frac{5}{7} \text{ and}$$

$$P(\overline{E}_3) = \left(1 - \frac{3}{8}\right) = \frac{5}{8}.$$

$\therefore$ P(none solves the problem)

$$= P[(\text{not } E_1) \text{ and } (\text{not } E_2) \text{ and } (\text{not } E_3)]$$
$$= P(\overline{E}_1 \cap \overline{E}_2 \cap \overline{E}_3)$$
$$= P(\overline{E}_1) \times P(\overline{E}_2) \times P(\overline{E}_3) \qquad [\because\ \overline{E}_1, \overline{E}_2, \overline{E}_3 \text{ are independent}]$$
$$= \left(\frac{2}{3} \times \frac{5}{7} \times \frac{5}{8}\right) = \frac{25}{84}.$$

$\therefore$ P(that the problem is solved)

$$= 1 - P(\text{none solves the problem})$$
$$= \left(1 - \frac{25}{84}\right) = \frac{59}{84}.$$

Hence, the required probability is $\frac{59}{84}$.

EXAMPLE 16 *A problem in mathematics is given to three students whose chances of solving it correctly are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ respectively. What is the probability that only one of them solves it correctly?* **[CBSE 1999C]**

SOLUTION Let A, B, C be the given students and let E_1, E_2 and E_3 be the events that the problem is solved by A, B, C respectively. Then, $\overline{E}_1$, $\overline{E}_2$ and $\overline{E}_3$ are the events that the given problem is not solved by A, B, C respectively. Then,

$$P(E_1) = \frac{1}{2};\ P(E_2) = \frac{1}{3};\ P(E_3) = \frac{1}{4};$$

$$P(\overline{E}_1) = \left(1 - \frac{1}{2}\right) = \frac{1}{2};\ P(\overline{E}_2) = \left(1 - \frac{1}{3}\right) = \frac{2}{3} \text{ and } P(\overline{E}_3) = \left(1 - \frac{1}{4}\right) = \frac{3}{4}.$$

P(exactly one of them solves the problem)

$$= P[(E_1 \cap \overline{E}_2 \cap \overline{E}_3) \text{ or } (\overline{E}_1 \cap E_2 \cap \overline{E}_3) \text{ or } (\overline{E}_1 \cap \overline{E}_2 \cap E_3)]$$
$$= P(E_1 \cap \overline{E}_2 \cap \overline{E}_3) + P(\overline{E}_1 \cap E_2 \cap \overline{E}_3) + P(\overline{E}_1 \cap \overline{E}_2 \cap E_3)$$

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$$= \{P(E_1) \times P(\overline{E}_2) \times P(\overline{E}_3)\} + \{P(\overline{E}_1) \times P(E_2) \times P(\overline{E}_3)\}$$
$$+ \{P(\overline{E}_1) \times P(\overline{E}_2) \times P(E_3)\}$$
$$= \left(\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}\right) + \left(\frac{1}{2} \times \frac{1}{3} \times \frac{3}{4}\right) + \left(\frac{1}{2} \times \frac{2}{3} \times \frac{1}{4}\right)$$
$$= \left(\frac{1}{4} + \frac{1}{8} + \frac{1}{12}\right) = \frac{11}{24}.$$

Hence, the required probability is $\frac{11}{24}$.

EXAMPLE 17 *Three critics review a book. For the three critics, the odds in favour of the book are* $(5:2)$, $(4:3)$ *and* $(3:4)$ *respectively. Find the probability that the majority is in favour of the book.*

SOLUTION Let A, B, C denote the events that the book be favoured by the first, second and third critic respectively. Then,

$$P(A) = \frac{5}{7}; P(B) = \frac{4}{7}; P(C) = \frac{3}{7};$$

$$P(\overline{A}) = \left(1 - \frac{5}{7}\right) = \frac{2}{7}; P(\overline{B}) = \left(1 - \frac{4}{7}\right) = \frac{3}{7} \text{ and } P(\overline{C}) = \left(1 - \frac{3}{7}\right) = \frac{4}{7}.$$

Required probability

$= P(2$ critics favour the book or 3 critics favour the book$)$

$= P(2$ critics favour the book$) + P(3$ critics favour the book$)$

$= P[\{A$ and B and not $C\}$ or $\{A$ and C and not $B\}$

or $\{B$ and C and not $A\}] + P(A$ and B and $C)$

$$= P(A \cap B \cap \overline{C}) + P(A \cap \overline{B} \cap C) + P(\overline{A} \cap B \cap C) + P(A \cap B \cap C)$$
$$= \{P(A) \times P(B) \times P(\overline{C})\} + \{P(A) \times P(\overline{B}) \times P(C)\}$$
$$+ \{P(\overline{A}) \times P(B) \times P(C)\} + \{P(A) \times P(B) \times P(C)\}$$
$$= \left(\frac{5}{7} \times \frac{4}{7} \times \frac{4}{7}\right) + \left(\frac{5}{7} \times \frac{3}{7} \times \frac{3}{7}\right) + \left(\frac{2}{7} \times \frac{4}{7} \times \frac{3}{7}\right) + \left(\frac{5}{7} \times \frac{4}{7} \times \frac{3}{7}\right)$$
$$= \left(\frac{80}{343} + \frac{45}{343} + \frac{24}{343} + \frac{60}{343}\right) = \frac{209}{343}.$$

Hence, the required probability is $\frac{209}{343}$.

EXAMPLE 18 *The odds against a man who is* 45 *years old, living till he is* 70 *are* $7:5$, *and the odds against his wife who is now* 36, *living till she is* 61 *are* $5:3$. *Find the probability that*

(i) the couple will be alive 25 *years hence*

(ii) at least one of them will be alive 25 *years hence.* **[CBSE 2007]**

SOLUTION Let E_1 = event that the husband will be alive 25 years hence, and

E_2 = event that the wife will be alive 25 years hence.

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Then, $P(E_1) = \frac{5}{12}$ and $P(E_2) = \frac{3}{8}$.

$\therefore \quad P(\overline{E}_1) = \left(1 - \frac{5}{12}\right) = \frac{7}{12}$ and $P(\overline{E}_2) = \left(1 - \frac{3}{8}\right) = \frac{5}{8}$.

Clearly, E_1 and E_2 are independent events.

(i) P(the couple will be alive 25 years hence)

$= P(E_1 \text{ and } E_2) = P(E_1 \cap E_2)$

$= P(E_1) \cdot P(E_2) = \left(\frac{5}{12} \times \frac{3}{8}\right) = \frac{5}{32}$.

(ii) P(at least one of them will be alive 25 years hence)

$= 1 - P$ (none will be alive 25 years hence)

$= 1 - P[(\text{not } E_1) \text{ and } (\text{not } E_2)]$

$= 1 - P(\overline{E}_1 \cap \overline{E}_2)$

$= 1 - [P(\overline{E}_1) \cdot P(\overline{E}_2)]$ [$\because$ $\overline{E}_1$ and $\overline{E}_2$ are independent]

$= 1 - \left(\frac{7}{12} \times \frac{5}{8}\right) = \left(1 - \frac{35}{96}\right) = \frac{61}{96}$.

EXAMPLE 19 *A, B and C shoot to hit a target. If* A *hits the target* 4 *times in* 5 *trials;* B *hits it* 3 *times in* 4 *trials and* C *hits it* 2 *times in* 3 *trials, what is the probability that the target is hit by at least* 2 *persons?*

SOLUTION Let E_1, E_2 and E_3 be the events that A hits the target, B hits the target and C hits the target respectively. Then,

$P(E_1) = \frac{4}{5}$, $P(E_2) = \frac{3}{4}$, $P(E_3) = \frac{2}{3}$;

$P(\overline{E}_1) = \left(1 - \frac{4}{5}\right) = \frac{1}{5}$, $P(\overline{E}_2) = \left(1 - \frac{3}{4}\right) = \frac{1}{4}$ and $P(\overline{E}_3) = \left(1 - \frac{2}{3}\right) = \frac{1}{3}$.

Case I A, B, C *all hit the target*

In this case, P(A, B and C all hit the target)

$= P(E_1 \text{ and } E_2 \text{ and } E_3)$

$= P(E_1) \cdot P(E_2) \cdot P(E_3)$ [$\because$ E_1, E_2, E_3 are independent]

$= \left(\frac{4}{5} \times \frac{3}{4} \times \frac{2}{3}\right) = \frac{2}{5}$.

Case II A *and* B *hit but not* C

In this case, P(A and B hit but not C)

$= P(E_1 \text{ and } E_2 \text{ and not } E_3)$

$= P(E_1 \cap E_2 \cap \overline{E}_3)$

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$= P(E_1) \cdot P(E_2) \cdot P(\overline{E}_3)$ [$\because$ $E_1, E_2, \overline{E}_3$ are independent]

$= \left(\frac{4}{5} \times \frac{3}{4} \times \frac{1}{3}\right) = \frac{1}{5}.$

Case III A *and* C *both hit but not* B

In this case, P(A and C hit but not B)

$= P(E_1 \text{ and } E_3 \text{ and } \overline{E}_2)$

$= P(E_1) \cdot P(E_3) \cdot P(\overline{E}_2)$ [$\because$ $E_1, E_3, \overline{E}_2$ are independent]

$= \left(\frac{4}{5} \times \frac{2}{3} \times \frac{1}{4}\right) = \frac{2}{15}.$

Case IV B *and* C *both hit but not* A

In this case, P(B and C hit but not A)

$= P(E_2 \text{ and } E_3 \text{ and } \overline{E}_1)$

$= P(E_2) \cdot P(E_3) \cdot P(\overline{E}_1)$ [$\because$ $E_2, E_3, \overline{E}_1$ are independent]

$= \left(\frac{3}{4} \times \frac{2}{3} \times \frac{1}{5}\right) = \frac{1}{10}.$

Clearly, all these are mutually exclusive.

Hence, required probability $= \left(\frac{2}{5} + \frac{1}{5} + \frac{2}{15} + \frac{1}{10}\right) = \frac{5}{6}.$

EXERCISE 29B

1. A bag contains 17 tickets, numbered from 1 to 17. A ticket is drawn and then another ticket is drawn without replacing the first one. Find the probability that both the tickets may show even numbers.
2. Two marbles are drawn successively from a box containing 3 black and 4 white marbles. Find the probability that both the marbles are black, if the first marble is not replaced before the second draw.
3. A card is drawn from a well-shuffled deck of 52 cards and without replacing this card, a second card is drawn. Find the probability that the first card is a club and the second card is a spade.
4. There is a box containing 30 bulbs of which 5 are defective. If two bulbs are chosen at random from the box in succession without replacing the first, what is the probability that both the bulbs chosen are defective?
5. A bag contains 10 white and 15 black balls. Two balls are drawn in succession without replacement. What is the probability that the first ball is white and the second is black?
6. An urn contains 5 white and 8 black balls. Two successive drawings of 3 balls at a time are made such that the balls drawn in the first draw are not replaced before the second draw. Find the probability that the first draw gives 3 white balls and the second draw gives 3 black balls.

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7. Let E_1 and E_2 be the events such that $P(E_1) = \frac{1}{3}$ and $P(E_2) = \frac{3}{5}$.

   Find:

   (i) $P(E_1 \cup E_2)$, when E_1 and E_2 are mutually exclusive,

   (ii) $P(E_1 \cap E_2)$, when E_1 and E_2 are independent.

8. If E_1 and E_2 are the two events such that $P(E_1) = \frac{1}{4}$, $P(E_2) = \frac{1}{3}$ and $P(E_1 \cup E_2) = \frac{1}{2}$, show that E_1 and E_2 are independent events.

9. If E_1 and E_2 are independent events such that $P(E_1) = 0.3$ and $P(E_2) = 0.4$, find (i) $P(E_1 \cap E_2)$ (ii) $P(E_1 \cap E_2)$ (iii) $P(\bar{E}_1 \cap \bar{E}_2)$ (iv) $P(\bar{E}_1 \cap E_2)$.

10. Let A and B be the events such that

    $P(A) = \frac{1}{2}$, $P(B) = \frac{7}{12}$ and $P(\text{not } A \text{ or not } B) = \frac{1}{4}$.

    State whether A and B are

    (i) mutually exclusive, (ii) independent.

11. Kamal and Vimal appeared for an interview for two vacancies. The probability of Kamal's selection is $\frac{1}{3}$ and that of Vimal's selection is $\frac{1}{5}$. Find the probability that only one of them will be selected.

12. Arun and Ved appeared for an interview for two vacancies. The probability of Arun's selection is $\frac{1}{4}$ and that of Ved's rejection is $\frac{2}{3}$. Find the probability that at least one of them will be selected.

13. A and B appear for an interview for two vacancies in the same post. The probability of A's selection is $\frac{1}{6}$ and that of B's selection is $\frac{1}{4}$. Find the probability that

    (i) both of them are selected

    (ii) only one of them is selected

    (iii) none is selected

    (iv) at least one of them is selected.

14. Given the probability that A can solve a problem is $\frac{2}{3}$, and the probability that B can solve the same problem is $\frac{3}{5}$, find the probability that

    (i) at least one of A and B will solve the problem

    (ii) none of the two will solve the problem.

15. A problem is given to three students whose chances of solving it are $\frac{1}{4}$, $\frac{1}{5}$ and $\frac{1}{6}$ respectively. Find the probability that the problem is solved.

16. The probabilities of A, B, C solving a problem are $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{6}$ respectively. If all the three try to solve the problem simultaneously, find the probability that exactly one of them will solve it.

17. A can hit a target 4 times in 5 shots, B can hit 3 times in 4 shots, and C can hit 2 times in 3 shots. Calculate the probability that

    (i) A, B and C all hit the target

    (ii) B and C hit and A does not hit the target.

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18. Neelam has offered physics, chemistry and mathematics in Class XII. She estimates that her probabilities of receiving a grade A in these courses are 0.2, 0.3 and 0.9 respectively. Find the probabilities that Neelam receives
(i) all A grades (ii) no A grade (iii) exactly 2 A grades.

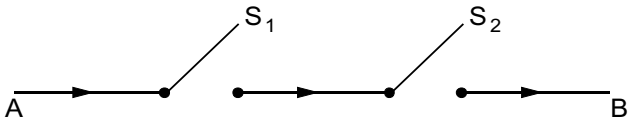
19. An article manufactured by a company consists of two parts X and Y. In the process of manufacture of part X, 8 out of 100 parts may be defective. Similarly, 5 out of 100 parts of Y may be defective. Calculate the probability that the assembled product will not be defective.

20. A town has two fire-extinguishing engines, functioning independently. The probability of availability of each engine when needed is 0.95. What is the probability that
(i) neither of them is available when needed?
(ii) an engine is available when needed?

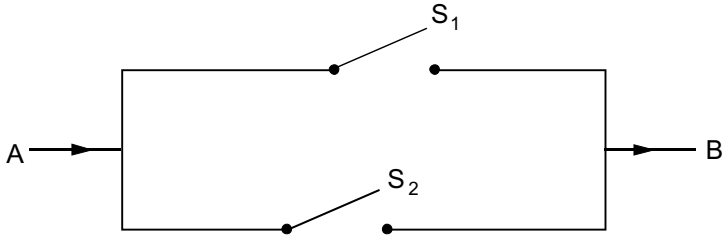
21. A machine operates only when all of its three components function. The probabilities of the failures of the first, second and third components are 0.14, 0.10 and 0.05 respectively. What is the probability that the machine will fail? **[CBSE 2005C]**

22. An anti-aircraft gun can take a maximum of 4 shots at an enemy plane moving away from it. The probabilities of hitting the plane at the first, second, third and fourth shots are 0.4, 0.3, 0.2 and 0.1 respectively. What is the probability that at least one shot hits the plane?

23. Let S_1 and S_2 be the two switches and let their probabilities of working be given by $P(S_1) = \frac{4}{5}$ and $P(S_2) = \frac{9}{10}$. Find the probability that the current flows from the terminal A to terminal B when S_1 and S_2 are installed in series, shown as follows:



24. Let S_1 and S_2 be two the switches and let their probabilities of working be given by $P(S_1) = \frac{2}{3}$ and $P(S_2) = \frac{3}{4}$. Find the probability that the current flows from terminal A to terminal B, when S_1 and S_2 are installed in parallel, as shown below:



25. A coin is tossed. If a head comes up, a die is thrown but if a tail comes up, the coin is tossed again. Find the probability of obtaining
(i) two tails (ii) a head and the number 6
(iii) a head and an even number.

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ANSWERS (EXERCISE 29B)

1. $\frac{7}{34}$ **2.** $\frac{1}{7}$ **3.** $\frac{13}{204}$ **4.** $\frac{2}{87}$ **5.** $\frac{1}{4}$ **6.** $\frac{7}{429}$ **7.** (i) $\frac{14}{15}$ (ii) $\frac{1}{5}$

9. (i) 0.12 (ii) 0.58 (iii) 0.42 (iv) 0.28 **10.** (i) No (ii) No **11.** $\frac{2}{5}$ **12.** $\frac{1}{2}$

13. (i) $\frac{1}{24}$ (ii) $\frac{1}{3}$ (iii) $\frac{5}{8}$ (iv) $\frac{3}{8}$ **14.** (i) $\frac{13}{15}$ (ii) $\frac{2}{15}$ **15.** $\frac{1}{2}$ **16.** $\frac{31}{72}$

17. (i) $\frac{2}{5}$ (ii) $\frac{1}{10}$ **18.** (i) 0.054 (ii) 0.056 (iii) 0.348 **19.** $\frac{437}{500}$

20. (i) $\frac{1}{400}$ (ii) $\frac{19}{200}$ **21.** 0.2647 **22.** 0.6976 **23.** $\frac{18}{25}$ **24.** $\frac{11}{12}$

25. (i) $\frac{1}{8}$ (ii) $\frac{1}{8}$ (iii) $\frac{3}{8}$

HINTS TO SOME SELECTED QUESTIONS (EXERCISE 29B)

6. Required probability $= \left\{\frac{{}^5C_3}{{}^{13}C_3} \times \frac{{}^8C_3}{{}^{10}C_3}\right\} = \left(\frac{5\times4\times3}{13\times12\times11} \times \frac{8\times7\times6}{10\times9\times8}\right)$

10. $P(\text{not } A \text{ or not } B) = P(\overline{A} \text{ or } \overline{B}) = P(\overline{A} \cup \overline{B})$

$= P(\overline{A \cap B}) = 1 - P(A \cap B)$

$\Rightarrow\ 1 - P(A \cap B) = \frac{1}{4} \Rightarrow\ P(A \cap B) = \frac{3}{4}.$

(i) Since $P(A \cap B) \neq 0$, so A and B are not mutually exclusive.

(ii) $P(A) \times P(B) = \left(\frac{1}{2} \times \frac{7}{12}\right) = \frac{7}{24} \neq P(A \cap B)$

$\Rightarrow$ A and B are not independent.

11. Let E_1 = event that Kamal is selected,
and E_2 = event that Vimal is selected. Then,

$P(E_1) = \frac{1}{3}, P(E_2) = \frac{1}{5}, P(\overline{E}_1) = \left(1 - \frac{1}{3}\right) = \frac{2}{3}$ and $P(\overline{E}_2) = \left(1 - \frac{1}{5}\right) = \frac{4}{5}.$

$\therefore$ required probability

$= P[(E_1 \text{ and not } E_2) \text{ or } (E_2 \text{ and not } E_1)]$

$= P[(E_1 \text{ and } \overline{E}_2) \text{ or } (E_2 \text{ and } \overline{E}_1)]$

$= P(E_1 \cap \overline{E}_2) + P(E_2 \cap \overline{E}_1)$

$= \{P(E_1) \times P(\overline{E}_2)\} + \{P(E_2) \times P(\overline{E}_1)\}.$

12. Let E_1 = event that Arun is selected,
and E_2 = event that Ved is selected.

Then, $P(E_1) = \frac{1}{4}$ and $P(E_2) = \left(1 - \frac{2}{3}\right) = \frac{1}{3}.$

Clearly, E_1 and E_2 are independent events.

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$\therefore \quad P(E_1 \cap E_2) = P(E_1) \times P(E_2) = \left(\frac{1}{4} \times \frac{1}{3}\right) = \frac{1}{12}.$

Required probability $= P(E_1 \text{ or } E_2) = P(E_1 \cup E_2)$

$= P(E_1) + P(E_2) - P(E_1 \cap E_2) = \left(\frac{1}{4} + \frac{1}{3} - \frac{1}{12}\right) = \frac{6}{12} = \frac{1}{2}.$

13. We have $P(A) = \frac{1}{6}, P(B) = \frac{1}{4}.$

$\therefore \quad P(\overline{A}) = 1 - P(A) = \left(1 - \frac{1}{6}\right) = \frac{5}{6}$ and $P(\overline{B}) = 1 - P(B) = \left(1 - \frac{1}{4}\right) = \frac{3}{4}.$

(i) P(both are selected)

$= P(A \cap B) = P(A) \times P(B).$

(ii) P(only one of them is selected)

$= P[(A \text{ and not } B) \text{ or } (B \text{ and not } A)]$

$= P(A \cap \overline{B}) + P(B \cap \overline{A})$

$= \{P(A) \times P(\overline{B})\} + \{P(B) \times P(\overline{A})\}.$

(iii) P(none is selected) $= P$(not A and not B)

$= P(\overline{A} \cap \overline{B}) = P(\overline{A}) \times P(\overline{B}).$

(iv) P(at least one is selected) $= 1 - P$(none is selected).

14. Let E_1 = event that A can solve the problem,
and E_2 = event that B can solve the problem.
Then, E_1 and E_2 are clearly independent events.

$\therefore \quad P(E_1 \cap E_2) = P(E_1) \times P(E_2) = \left(\frac{2}{3} \times \frac{3}{5}\right) = \frac{2}{5}.$

(i) P(at least one of A and B can solve the problem)

$= P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2) = \left(\frac{2}{3} + \frac{3}{5} - \frac{2}{5}\right) = \frac{13}{15}.$

(ii) P(none can solve the problem)

$= 1 - P$(at least one can solve the problem)

$= \left(1 - \frac{13}{15}\right) = \frac{2}{15}.$

15. Let A, B, C be the events of solving the problem by the 1st, 2nd and 3rd student respectively. Then,

$P(A) = \frac{1}{4}, P(B) = \frac{1}{5}$ and $P(C) = \frac{1}{6}$

$\Rightarrow \quad P(\overline{A}) = \left(1 - \frac{1}{4}\right) = \frac{3}{4}; P(\overline{B}) = \left(1 - \frac{1}{5}\right) = \frac{4}{5}$ and $P(\overline{C}) = \left(1 - \frac{1}{6}\right) = \frac{5}{6}.$

$\therefore \quad P$(none solves the problem) $= P[(\text{not } A) \text{ and } (\text{not } B) \text{ and } (\text{not } C)]$

$= P(\overline{A} \text{ and } \overline{B} \text{ and } \overline{C}) = P(\overline{A}) \times P(\overline{B}) \times P(\overline{C}).$

16. Given $P(A) = \frac{1}{3}, P(B) = \frac{1}{4}$ and $P(C) = \frac{1}{6}$

$\Rightarrow \quad P(\overline{A}) = \left(1 - \frac{1}{3}\right) = \frac{2}{3}, P(\overline{B}) = \left(1 - \frac{1}{4}\right) = \frac{3}{4}$ and $P(\overline{C}) = \left(1 - \frac{1}{6}\right) = \frac{5}{6}.$

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$\therefore$ required probability

$= P$(exactly one of them solves the problem)

$= P[(A \cap \overline{B} \cap \overline{C})$ or $(\overline{A} \cap B \cap \overline{C})$ or $(\overline{A} \cap \overline{B} \cap C)]$

$= P(A \cap \overline{B} \cap \overline{C}) + P(\overline{A} \cap B \cap \overline{C}) + P(\overline{A} \cap \overline{B} \cap C)$

$= \{P(A) \times P(\overline{B}) \times P(\overline{C})\} + \{P(\overline{A}) \times P(B) \times P(\overline{C})\} + \{P(\overline{A}) \times P(\overline{B}) \times P(C)\}.$

17. $P(\text{A hits}) = \frac{4}{5}$, $P(\text{B hits}) = \frac{3}{4}$ and $P(\text{C hits}) = \frac{2}{3}$.

(i) P(A hits and B hits and C hits) $= P(\text{A}) \times P(\text{B}) \times P(\text{C})$.

(ii) P(B hits and C hits and A does not hit)

$= P(\text{B}) \times P(\text{C}) \times P(\overline{\text{A}})$.

18. (i) P(getting all A grades)

$= P(A_1\ A_2\ A_3) = P(A_1) \times P(A_2) \times P(A_3)$.

(ii) P(getting no A grade)

$= P(\overline{A}_1\ \overline{A}_2\ \overline{A}_3) = P(\overline{A}_1) \times P(\overline{A}_2) \times P(\overline{A}_3)$.

(iii) P(getting exactly two A grades)

$= P[(\overline{A}_1\ A_2\ A_3)$ or $(A_1\ \overline{A}_2\ A_3)$ or $(A_1\ A_2\ \overline{A}_3)]$

$= P(\overline{A}_1\ A_2\ A_3) + P(A_1\ \overline{A}_2\ A_3) + P(A_1\ A_2\ \overline{A}_3)$

$= \{P(\overline{A}_1) \times P(A_2) \times P(A_3)\} + \{P(A_1) \times P(\overline{A}_2) \times P(A_3)\}$

$+ \{P(A_1) \times P(A_2) \times P(\overline{A}_3)\}.$

19. $P(X \text{ is defective}) = \frac{8}{100} = \frac{2}{25}$.

$P(Y \text{ is defective}) = \frac{5}{100} = \frac{1}{20}$.

$P(X \text{ is not defective}) = \left(1 - \frac{2}{25}\right) = \frac{23}{25}$.

$P(Y \text{ is not defective}) = \left(1 - \frac{1}{20}\right) = \frac{19}{20}$.

Required probability $= P$(assembled part is not defective)

$= P(X \text{ is not defective and } Y \text{ is not defective})$

$= \left(\frac{23}{25} \times \frac{19}{20}\right) = \frac{437}{500}$.

20. Let E_1 = event of availability of the first engine.

And, E_2 = event of availability of the second engine.

Then, $P(E_1) = P(E_2) = 0.95$ and $P(\overline{E}_1) = P(\overline{E}_2) = (1 - 0.95) = 0.05$.

(i) P(neither of them is available when needed)

$= P(\overline{E}_1 \text{ and } \overline{E}_2) = P(\overline{E}_1) \times P(\overline{E}_2)$.

(ii) P(an engine is available when needed)

$= P[(E_1 \text{ and not } E_2)$ or $(E_2 \text{ and not } E_1)]$

$= P[(E_1 \text{ and } \overline{E}_2)$ or $(E_2 \text{ and } \overline{E}_1)]$

$= P(E_1 \cap \overline{E}_2) + P(E_2 \cap \overline{E}_1)$

$= \{P(E_1) \times P(\overline{E}_2)\} + \{P(E_2) \times P(\overline{E}_1)\}.$

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21. Let E_1, E_2, E_3 be the respective events that the 1st, 2nd and 3rd components function. Then,

$P(\overline{E}_1) = 0.14, P(\overline{E}_2) = 0.10$ and $P(\overline{E}_3) = 0.05$

$\Rightarrow \quad P(E_1) = (1 - 0.14) = 0.86, P(E_2) = (1 - 0.10) = 0.90$ and $P(E_3) = (1 - 0.05) = 0.95$

$\Rightarrow \quad P(\text{machine fails}) = 1 - P(\text{machine functions})$

$= 1 - P[(E_1 \text{ and } E_2 \text{ and } E_3)]$

$= 1 - [P(E_1) \times P(E_2) \times P(E_3)].$

22. Let E_1, E_2, E_3, E_4 be the respective events that the plane is hit in the 1st, 2nd, 3rd and 4th shot. Then,

$P(E_1) = 0.4, P(E_2) = 0.3, P(E_3) = 0.2$ and $P(E_4) = 0.1$

$\therefore \quad P(\overline{E}_1) = (1 - 0.4) = 0.6, P(\overline{E}_2) = (1 - 0.3) = 0.7,$

$P(\overline{E}_3) = (1 - 0.2) = 0.8, P(\overline{E}_4) = (1 - 0.1) = 0.9.$

$\therefore \quad P(\text{at least one shot hits the plane}) = 1 - P(\overline{E}_1 \text{ and } \overline{E}_2 \text{ and } \overline{E}_3 \text{ and } \overline{E}_4)$

$= 1 - \{P(\overline{E}_1) \times P(\overline{E}_2) \times P(\overline{E}_3) \times P(\overline{E}_4)\}.$

23. $P(\text{current flows from A to B})$

$= P(S_1 \text{ is closed and } S_2 \text{ is closed})$

$= P(S_1 \text{ and } S_2) = P(S_1) \times P(S_2).$

24. $P(\text{the current flows}) = P(S_1 \text{ or } S_2)$

$= P(S_1 \cup S_2) = P(S_1) + P(S_2) - P(S_1 \cap S_2)$

$= P(S_1) + P(S_2) - P(S_1) \times P(S_2).$

25. $S = \{H1, H2, H3, H4, H5, H6, TT, TH\} \rightarrow n(S) = 8.$

(i) $P(\text{two tails}) = \frac{1}{8}.$

(ii) $P(\text{head and the number 6}) = \frac{1}{8}.$

(iii) $P(\text{head and an even number}) = \frac{3}{8}.$

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30. BAYES'S THEOREM AND ITS APPLICATIONS

Theorem of Total Probability

THEOREM *Let $E_1, E_2, \ldots, E_n$ be mutually exclusive and exhaustive events associated with a random experiment and let E be an event that occurs with some E_i. Then, prove that*

$$P(E) = \sum_{i=1}^{n} P(E/E_i) \cdot P(E_i).$$

PROOF Let S be the sample space. Then,

$$S = E_1 \cup E_2 \cup \ldots \cup E_n \text{ and } E_i \cap E_j = \phi \text{ for } i \neq j.$$

$$\therefore \quad E = E \cap S = E \cap (E_1 \cup E_2 \cup \ldots \cup E_n)$$

$$= (E \cap E_1) \cup (E \cap E_2) \cup \ldots \cup (E \cap E_n)$$

$$\Rightarrow \quad P(E) = P\{(E \cap E_1) \cup (E \cap E_2) \cup \ldots \cup (E \cap E_n)\}$$

$$= P(E \cap E_1) + P(E \cap E_2) + \ldots + P(E \cap E_n)$$

$\{\because (E \cap E_1), (E \cap E_2), \ldots, (E \cap E_n)$ are pairwise disjoint$\}$

$$= P(E/E_1) \cdot P(E_1) + P(E/E_2) \cdot P(E_2) + \ldots + P(E/E_n) \cdot P(E_n)$$

[by multiplication theorem]

$$= \sum_{i=1}^{n} P(E/E_i) \cdot P(E_i).$$

EXAMPLE *There are three urns containing 3 white and 2 black balls; 2 white and 3 black balls; 1 black and 4 white balls respectively. There is equal probability of each urn being chosen. One ball is drawn from an urn chosen at random. What is the probability that a white ball is drawn?*

SOLUTION Let E_1, E_2 and E_3 be the events of choosing the first, second and third urn respectively. Then,

$$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}.$$

Let E be the event that a white ball is drawn. Then,

$$P(E/E_1) = \frac{3}{5}, \; P(E/E_2) = \frac{2}{5} \text{ and } P(E/E_3) = \frac{4}{5}.$$

By the theorem of total probability, we have

$$P(E) = P(E/E_1) \cdot P(E_1) + P(E/E_2) \cdot P(E_2) + P(E/E_3) \cdot P(E_3)$$

$$= \left(\frac{3}{5} \times \frac{1}{3} + \frac{2}{5} \times \frac{1}{3} + \frac{4}{5} \times \frac{1}{3}\right) = \left(\frac{1}{5} + \frac{2}{15} + \frac{4}{15}\right) = \frac{9}{15} = \frac{3}{5}.$$

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BAYES'S THEOREM *Let $E_1, E_2, \ldots, E_n$ be mutually exclusive and exhaustive events, associated with a random experiment, and let E be any event that occurs with some E_i. Then,*

$$P(E_i/E) = \frac{P(E/E_i) \cdot P(E_i)}{\sum_{i=1}^{n} P(E/E_i) \cdot P(E_i)}; \; i = 1, 2, 3, \ldots, n.$$

PROOF By the theorem of total probability, we have

$$P(E) = \sum_{i=1}^{n} P(E/E_i) \cdot P(E_i) \qquad \ldots \text{(i)}$$

$$\therefore \quad P(E_i/E) = \frac{P(E \cap E_i)}{P(E)} \qquad \text{[by multiplication theorem]}$$

$$= \frac{P(E/E_i) \cdot P(E_i)}{P(E)} \qquad \left[\because \quad P(E/E_i) = \frac{P(E \cap E_i)}{P(E_i)} \right]$$

$$= \frac{P(E/E_i) \cdot P(E_i)}{\sum_{i=1}^{n} P(E/E_i) \cdot P(E_i)} \text{ [using (i)].}$$

Hence, $P(E_i/E) = \dfrac{P(E/E_i) \cdot P(E_i)}{\sum_{i=1}^{n} P(E/E_i) \cdot P(E_i)}$.

SOLVED EXAMPLES

EXAMPLE 1 *A factory has three machines, X, Y and Z, producing 1000, 2000 and 3000 bolts per day respectively. The machine X produces 1% defective bolts, Y produces 1.5% defective bolts and Z produces 2% defective bolts. At the end of the day, a bolt is drawn at random and it is found to be defective. What is the probability that this defective bolt has been produced by the machine X?* **[CBSE 2002]**

SOLUTION Total number of bolts produced in a day

$$= (1000 + 2000 + 3000) = 6000.$$

Let E_1, E_2 and E_3 be the events of drawing a bolt produced by machines X, Y and Z respectively. Then,

$$P(E_1) = \frac{1000}{6000} = \frac{1}{6}; \; P(E_2) = \frac{2000}{6000} = \frac{1}{3} \text{ and } P(E_3) = \frac{3000}{6000} = \frac{1}{2}.$$

Let E be the event of drawing a defective bolt. Then,

$P(E/E_1)$ = probability of drawing a defective bolt, given that it is produced by the machine X

$$= \frac{1}{100}.$$

$P(E/E_2)$ = probability of drawing a defective bolt, given that it is produced by the machine Y

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$$= \frac{1.5}{100} = \frac{15}{1000} = \frac{3}{200}.$$

$P(E/E_3)$ = probability of drawing a defective bolt, given that it is produced by the machine Z

$$= \frac{2}{100} = \frac{1}{50}.$$

Required probability

$= P(E_1/E)$

= probability that the bolt drawn is produced by X, given that it is defective

$$= \frac{P(E_1) \cdot P(E/E_1)}{P(E_1) \cdot P(E/E_1) + P(E_2) \cdot P(E/E_2) + P(E_3) \cdot P(E/E_3)}$$

$$= \frac{\left(\frac{1}{6} \times \frac{1}{100}\right)}{\left(\frac{1}{6} \times \frac{1}{100}\right) + \left(\frac{1}{3} \times \frac{3}{200}\right) + \left(\frac{1}{2} \times \frac{1}{50}\right)}$$

$$= \left(\frac{1}{600} \times \frac{600}{10}\right) = \frac{1}{10} = 0.1.$$

Hence, the required probability is 0.1.

EXAMPLE 2 *In a bolt factory, three machines, A, B, C, manufacture* 25%, 35% *and* 40% *of the total production respectively. Of their respective outputs,* 5%, 4% *and* 2% *are defective. A bolt is drawn at random from the total product and it is found to be defective. Find the probability that it was manufactured by the machine C.* **[CBSE 2006, '10]**

SOLUTION Let E_1, E_2 and E_3 be the events of drawing a bolt produced by machine A, B and C respectively. Then,

$$P(E_1) = \frac{25}{100} = \frac{1}{4}, \quad P(E_2) = \frac{35}{100} = \frac{7}{20}, \text{ and } P(E_3) = \frac{40}{100} = \frac{2}{5}.$$

Let E be the event of drawing a defective bolt. Then,

$P(E/E_1)$ = probability of drawing a defective bolt, given that it is produced by the machine A

$$= \frac{5}{100} = \frac{1}{20}.$$

$P(E/E_2)$ = probability of drawing a defective bolt, given that it is produced by the machine B

$$= \frac{4}{100} = \frac{1}{25}.$$

$P(E/E_3)$ = probability of drawing a defective bolt, given that it is produced by the machine C

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$$= \frac{2}{100} = \frac{1}{50}.$$

Probability that the bolt drawn is manufactured by C, given that it is defective

$$= P(E_3/E)$$

$$= \frac{P(E/E_3) \cdot P(E_3)}{P(E/E_1) \cdot P(E_1) + P(E/E_2) \cdot P(E_2) + P(E/E_3) \cdot P(E_3)}$$

[by Bayes's theorem]

$$= \frac{\left(\frac{1}{50} \times \frac{2}{5}\right)}{\left(\frac{1}{20} \times \frac{1}{4}\right) + \left(\frac{1}{25} \times \frac{7}{20}\right) + \left(\frac{1}{50} \times \frac{2}{5}\right)} = \left(\frac{1}{125} \times \frac{2000}{69}\right) = \frac{16}{69}.$$

Hence, the required probability is $\frac{16}{69}$.

EXAMPLE 3 *A company has two plants to manufacture bicycles. The first plant manufactures 60% of the bicycles and the second plant, 40%. Also, 80% of the bicycles are rated of standard quality at the first plant and 90% of standard quality at the second plant. A bicycle is picked up at random and found to be of standard quality. Find the probability that it comes from the second plant.* **[CBSE 2003]**

SOLUTION Let E_1 and E_2 be the events of choosing a bicycle from the first plant and the second plant respectively. Then,

$$P(E_1) = \frac{60}{100} = \frac{3}{5}, \text{ and } P(E_2) = \frac{40}{100} = \frac{2}{5}.$$

Let E be the event of choosing a bicycle of standard quality. Then,

$P(E/E_1)$ = probability of choosing a bicycle of standard quality, given that it is produced by the first plant

$$= \frac{80}{100} = \frac{4}{5}.$$

$P(E/E_2)$ = probability of choosing a bicycle of standard quality, given that it is produced by the second plant

$$= \frac{90}{100} = \frac{9}{10}.$$

The required probability

$P(E_2/E)$ = probability of choosing a bicycle from the second plant, given that it is of standard quality

$$= \frac{P(E_2) \cdot P(E/E_2)}{P(E_1) \cdot P(E/E_1) + P(E_2) \cdot P(E/E_2)}$$ [by Bayes's theorem]

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$$= \frac{\left(\frac{2}{5} \times \frac{9}{10}\right)}{\left(\frac{3}{5} \times \frac{4}{5}\right) + \left(\frac{2}{5} \times \frac{9}{10}\right)} = \frac{3}{7}.$$

EXAMPLE 4 *An insurance company insured* 2000 *scooter drivers,* 4000 *car drivers and* 6000 *truck drivers. The probability of an accident involving a scooter, a car and a truck is* $\frac{1}{100}$, $\frac{3}{100}$ *and* $\frac{3}{20}$ *respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?* **[CBSE 2000, '02, '08]**

SOLUTION Total number of persons insured $= (2000 + 4000 + 6000) = 12000.$

Let E_1, E_2 and E_3 be the events of choosing a scooter driver, a car driver and a truck driver respectively. Then,

$$P(E_1) = \frac{2000}{12000} = \frac{1}{6}, P(E_2) = \frac{4000}{12000} = \frac{1}{3} \text{ and } P(E_3) = \frac{6000}{12000} = \frac{1}{2}.$$

Let E be the event of an insured person meeting with an accident. Then,

$P(E/E_1)$ = probability that an insured person meets with an accident, given that he is a scooter driver

$$= \frac{1}{100}.$$

Similarly, $P(E/E_2) = \frac{3}{100}$ and $P(E/E_3) = \frac{3}{20}.$

Required probability

$= P(E_1/E)$ [by Bayes's theorem]

= probability of choosing a scooter driver, given that he meets with an accident

$$= \frac{P(E/E_1) \cdot P(E_1)}{P(E/E_1) \cdot P(E_1) + P(E/E_2) \cdot P(E_2) + P(E/E_3) \cdot P(E_3)}$$

$$= \frac{\left(\frac{1}{100} \times \frac{1}{6}\right)}{\left(\frac{1}{100} \times \frac{1}{6}\right) + \left(\frac{3}{100} \times \frac{1}{3}\right) + \left(\frac{3}{20} \times \frac{1}{2}\right)} = \frac{1}{52}.$$

Hence, the required probability is $\frac{1}{52}.$

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EXAMPLE 5 *A doctor is to visit a patient. From past experience, it is known that the probabilities that he will come by train, bus, scooter or by car are respectively* $\frac{3}{10}$, $\frac{1}{5}$, $\frac{1}{10}$ *and* $\frac{2}{5}$. *The probabilities that he will be late are* $\frac{1}{4}$, $\frac{1}{3}$ *and* $\frac{1}{12}$, *if he comes by train, bus and scooter respectively; but if he comes by car, he will not be late. When he arrives, he is late. What is the probability that he has come by train?* **[CBSE 2008C]**

SOLUTION Let E_1, E_2, E_3 and E_4 be the events that the doctor comes by train, bus, scooter and car respectively. Then,

$$P(E_1) = \frac{3}{10}, \quad P(E_2) = \frac{1}{5}, \quad P(E_3) = \frac{1}{10} \text{ and } P(E_4) = \frac{2}{5}.$$

Let E be the event that the doctor is late. Then,

$P(E/E_1)$ = probability that the doctor is late, given that he comes by train

$$= \frac{1}{4}.$$

$P(E/E_2)$ = probability that the doctor is late, given that he comes by bus

$$= \frac{1}{3}.$$

$P(E/E_3)$ = probability that the doctor is late, given that he comes by scooter

$$= \frac{1}{12}.$$

$P(E/E_4)$ = probability that the doctor is late, given that he comes by car

$$= 0.$$

Probability that he comes by train, given that he is late

$$= P(E_1/E)$$

$$= \frac{P(E_1) \cdot P(E/E_1)}{P(E_1) \cdot P(E/E_1) + P(E_2) \cdot P(E/E_2) + P(E_3) \cdot P(E/E_3) + P(E_4) \cdot P(E/E_4)}$$

[by Bayes's theorem]

$$= \frac{\left(\frac{3}{10} \times \frac{1}{4}\right)}{\left(\frac{3}{10} \times \frac{1}{4}\right) + \left(\frac{1}{5} \times \frac{1}{3}\right) + \left(\frac{1}{10} \times \frac{1}{12}\right) + \left(\frac{2}{5} \times 0\right)}$$

$$= \left(\frac{3}{40} \times \frac{120}{18}\right) = \frac{1}{2}.$$

Hence, the required probability is $\frac{1}{2}$.

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EXAMPLE 6 *A man is known to speak the truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.*

[CBSE 2005, '11]

SOLUTION In a throw of a die, let

E_1 = event of getting a six,

E_2 = event of not getting a six, and

E = event that the man reports that it is a six.

Then, $P(E_1) = \frac{1}{6}$, and $P(E_2) = \left(1 - \frac{1}{6}\right) = \frac{5}{6}$.

$P(E/E_1)$ = probability that the man reports that six occurs, when six has actually occurred

= probability that the man speaks the truth

$= \frac{3}{4}$.

$P(E/E_2)$ = probability that the man reports that six occurs, when six has not actually occurred

= probability that the man does not speak the truth

$= \left(1 - \frac{3}{4}\right) = \frac{1}{4}$.

Probability of getting a six, given that the man reports it to be six

$= P(E_1/E)$

$= \frac{P(E/E_1) \cdot P(E_1)}{P(E/E_1) \cdot P(E_1) + P(E/E_2) \cdot P(E_2)}$ [by Bayes's theorem]

$= \frac{\left(\frac{3}{4} \times \frac{1}{6}\right)}{\left(\frac{3}{4} \times \frac{1}{6}\right) + \left(\frac{1}{4} \times \frac{5}{6}\right)} = \left(\frac{1}{8} \times 3\right) = \frac{3}{8}$.

Hence, the required probability is $\frac{3}{8}$.

EXAMPLE 7 *In an examination, an examinee either guesses or copies or knows the answer to a multiple-choice question with four choices. The probability that he makes a guess is* (1/3) *and the probability that he copies the answer is* (1/6). *The probability that his answer is correct, given that he copied it, is* (1/8). *The probability that his answer is correct, given that he guessed it, is* (1/4). *Find the probability that he knew the answer to the question, given that he correctly answered it.*

SOLUTION Let E_1 = event that the examinee guesses the answer,

E_2 = event that he copies the answer,

E_3 = event that he knows the answer, and

E = event that he answers correctly.

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Then, $P(E_1) = \frac{1}{3}$, $P(E_2) = \frac{1}{6}$, and $P(E_3) = 1 - \left(\frac{1}{3} + \frac{1}{6}\right) = \frac{1}{2}$

[$\because$ E_1, E_2, E_3 are mutually exclusive and exhaustive].

$\therefore$ $P(E/E_1)$ = probability that he answers correctly, given that he guesses

$= \frac{1}{4}$.

$P(E/E_2)$ = probability that he answers correctly, given that he copies

$= \frac{1}{8}$.

$P(E/E_3)$ = probability that he answers correctly, given that he knew the answer

$= 1$.

Required probability

$$= P(E_3/E)$$

$$= \frac{P(E/E_3) \cdot P(E_3)}{P(E/E_1) \cdot P(E_1) + P(E/E_2) \cdot P(E_2) + P(E/E_3) \cdot P(E_3)}$$

[by Bayes's theorem]

$$= \frac{\left(1 \times \frac{1}{2}\right)}{\left(\frac{1}{4} \times \frac{1}{3}\right) + \left(\frac{1}{8} \times \frac{1}{6}\right) + \left(1 \times \frac{1}{2}\right)} = \frac{24}{29}.$$

Hence, the required probability is $\frac{24}{29}$.

EXAMPLE 8 *By examining the chest X-ray, the probability that a person is diagnosed with TB when he is actually suffering from it, is* 0.99. *The probability that the doctor incorrectly diagnoses a person to be having TB, on the basis of X-ray reports, is* 0.001. *In a certain city,* 1 *in* 1000 *persons suffers from TB. A person is selected at random and is diagnosed to have TB. What is the chance that he actually has TB?*

SOLUTION Let E = event that the doctor diagnoses TB,

E_1 = event that the person selected is suffering from TB, and

E_2 = event that the person selected is not suffering from TB.

Then, $P(E_1) = \frac{1}{1000}$ and $P(E_2) = \left(1 - \frac{1}{1000}\right) = \frac{999}{1000}$.

$P(E/E_1)$ = probability that TB is diagnosed, when the person actually has TB

$= \frac{99}{100}$.

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$P(E/E_2)$ = probability that TB is diagnosed, when the person has no TB

$$= \frac{1}{1000}.$$

Using Bayes's theorem, we have

$P(E_1/E)$ = probability of a person actually having TB, if it is known that he is diagnosed to have TB

$$= \frac{P(E/E_1) \cdot P(E_1)}{P(E/E_1) \cdot P(E_1) + P(E/E_2) \cdot P(E_2)}$$

$$= \frac{\left(\frac{99}{100} \times \frac{1}{1000}\right)}{\left(\frac{99}{100} \times \frac{1}{1000}\right) + \left(\frac{1}{1000} \times \frac{999}{1000}\right)} = \frac{110}{221}.$$

Hence, the required probability is $\frac{110}{221}$.

EXAMPLE 9 *Bag A contains 2 white and 3 red balls, and bag B contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that it was drawn from bag B.*

[CBSE 2004C]

SOLUTION Let E_1 = event of choosing bag A,

E_2 = event of choosing bag B, and

E = event of drawing a red ball.

Then, $P(E_1) = \frac{1}{2}$ and $P(E_2) = \frac{1}{2}$.

Also, $P(E/E_1)$ = event of drawing a red ball from bag $A = \frac{3}{5}$, and

$P(E/E_2)$ = event of drawing a red ball from bag $B = \frac{5}{9}$.

Probability of drawing a ball from B, it being given that it is red

$= P(E_2/E)$

$$= \frac{P(E/E_2) \cdot P(E_2)}{P(E/E_1) \cdot P(E_1) + P(E/E_2) \cdot P(E_2)} \quad \text{[by Bayes's theorem]}$$

$$= \frac{\left(\frac{5}{9} \times \frac{1}{2}\right)}{\left(\frac{3}{5} \times \frac{1}{2}\right) + \left(\frac{5}{9} \times \frac{1}{2}\right)} = \frac{25}{52}.$$

Hence, the required probability is $\frac{25}{52}$.

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EXAMPLE 10 *There are 5 bags, each containing 5 white balls and 3 black balls. Also, there are 6 bags, each containing 2 white balls and 4 black balls. A white ball is drawn at random. Find the probability that this white ball is from a bag of the first group.*

SOLUTION Let E_1 = event of selecting a bag from the first group,

E_2 = event of selecting a bag from the second group, and

E = event of drawing a white ball.

Then, $P(E_1) = \frac{5}{11}$ and $P(E_2) = \frac{6}{11}$.

$P(E/E_1)$ = probability of getting a white ball, given that it is from a bag of the first group

$= \frac{5}{8}$.

$P(E/E_2)$ = probability of getting a white ball, given that it is from a bag of the second group

$= \frac{2}{6} = \frac{1}{3}$.

Probability of getting the ball from a bag of the first group, given that it is white

$$= P(E_1/E)$$

$$= \frac{P(E/E_1)\cdot P(E_1)}{P(E/E_1)\cdot P(E_1) + P(E/E_2)\cdot P(E_2)} \text{[by Bayes's theorem]}$$

$$= \frac{\left(\frac{5}{8}\times\frac{5}{11}\right)}{\left(\frac{5}{8}\times\frac{5}{11}\right)+\left(\frac{1}{3}\times\frac{6}{11}\right)} = \frac{75}{123}.$$

EXAMPLE 11 *Urn A contains 1 white, 2 black and 3 red balls; urn B contains 2 white, 1 black and 1 red ball; and urn C contains 4 white, 5 black and 3 red balls. One urn is chosen at random and two balls are drawn. These happen to be one white and one red. What is the probability that they come from urn A?* **[CBSE 2009]**

SOLUTION Let E_1, E_2, E_3 be the events that the balls are drawn from urn A, urn B and urn C respectively, and let E be the event that the balls drawn are one white and one red. Then, $P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$.

$P(E/E_1)$ = probability that the balls drawn are one white and one red, given that the balls are from urn A

$$= \frac{{}^1C_1 \times {}^3C_1}{{}^6C_2} = \frac{3}{15} = \frac{1}{5}.$$

$P(E/E_2)$ = probability that the balls drawn are one white and one red, given that the balls are from urn B

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$$= \frac{{}^2C_1 \times {}^1C_1}{{}^4C_2} = \frac{2}{6} = \frac{1}{3}.$$

$P(E/E_3)$ = probability that the balls drawn are one white and one red, given that the balls are from urn C

$$= \frac{{}^4C_1 \times {}^3C_1}{{}^{12}C_2} = \frac{12}{66} = \frac{2}{11}.$$

Probability that the balls drawn are from urn A, it being given that the balls drawn are one white and one red

$$= P(E_1/E)$$

$$= \frac{P(E/E_1) \cdot P(E_1)}{P(E/E_1) \cdot P(E_1) + P(E/E_2) \cdot P(E_2) + P(E/E_3) \cdot P(E_3)}$$

[by Bayes's theorem]

$$= \frac{\left(\frac{1}{5} \times \frac{1}{3}\right)}{\left(\frac{1}{5} \times \frac{1}{3}\right) + \left(\frac{1}{3} \times \frac{1}{3}\right) + \left(\frac{2}{11} \times \frac{1}{3}\right)}$$

$$= \left(\frac{1}{15} \times \frac{495}{118}\right) = \frac{33}{118}.$$

Hence, the required probability is $\frac{33}{118}$.

EXAMPLE 12 *A card from a pack of* 52 *cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both spades. Find the probability of the lost card being a spade.* **[CBSE 2002]**

SOLUTION Let E_1, E_2, E_3 and E_4 be the events of losing a card of spades, clubs, hearts and diamonds respectively.

Then, $P(E_1) = P(E_2) = P(E_3) = P(E_4) = \frac{13}{52} = \frac{1}{4}$.

Let E be the event of drawing 2 spades from the remaining 51 cards. Then,

$P(E/E_1)$ = probability of drawing 2 spades, given that a card of spades is missing

$$= \frac{{}^{12}C_2}{{}^{51}C_2} = \frac{(12 \times 11)}{2!} \times \frac{2!}{(51 \times 50)} = \frac{22}{425}.$$

$P(E/E_2)$ = probability of drawing 2 spades, given that a card of clubs is missing

$$= \frac{{}^{13}C_2}{{}^{51}C_2} = \frac{(13 \times 12)}{2!} \times \frac{2!}{(51 \times 50)} = \frac{26}{425}.$$

$P(E/E_3)$ = probability of drawing 2 spades, given that a card of hearts is missing

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$$= \frac{^{13}C_2}{^{51}C_2} = \frac{26}{425}.$$

$P(E/E_4)$ = probability of drawing 2 spades, given that a card of diamonds is missing

$$= \frac{^{13}C_2}{^{51}C_2} = \frac{26}{425}.$$

$\therefore$ $P(E_1/E)$ = probability of the lost card being a spade, given that 2 spades are drawn from the remaining 51 cards

$$= \frac{P(E_1)\cdot P(E/E_1)}{P(E_1)\cdot P(E/E_1) + P(E_2)\cdot P(E/E_2) + P(E_3)\cdot P(E/E_3) + P(E_4)\cdot P(E/E_4)}$$

$$= \frac{\left(\frac{1}{4}\times\frac{22}{425}\right)}{\left(\frac{1}{4}\times\frac{22}{425}\right) + \left(\frac{1}{4}\times\frac{26}{425}\right) + \left(\frac{1}{4}\times\frac{26}{425}\right) + \left(\frac{1}{4}\times\frac{26}{425}\right)}$$

$$= \frac{22}{100} = 0.22.$$

Hence, the required probability is 0.22.

EXERCISE 30

Long-Answer Questions

1. In a bulb factory, three machines, A, B, C, manufacture 60%, 25% and 15% of the total production respectively. Of their respective outputs, 1%, 2% and 1% are defective. A bulb is drawn at random from the total product and it is found to be defective. Find the probability that it was manufactured by machine C.
2. A company manufactures scooters at two plants, A and B. Plant A produces 80% and plant B produces 20% of the total product. 85% of the scooters produced at plant A and 65% of the scooters produced at plant B are of standard quality. A scooter produced by the company is selected at random and it is found to be of standard quality. What is the probability that it was manufactured at plant A?
3. In a certain college, 4% of boys and 1% of girls are taller than 1.75 metres. Furthermore, 60% of the students are girls. If a student is selected at random and is taller than 1.75 metres, what is the probability that the selected student is a girl?
4. In a class, 5% of the boys and 10% of the girls have an IQ of more than 150. In this class, 60% of the students are boys. If a student is selected at random and found to have an IQ of more than 150, find the probability that the student is a boy.

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5. Suppose 5% of men and 0.25% of women have grey hair. A grey-haired person is selected at random. What is the probability of this person being male? Assume that there are equal number of males and females. **[CBSE 2011]**

6. Two groups are competing for the positions on the board of directors of a corporation. The probabilities that the first and the second groups will win are 0.6 and 0.4 respectively. Further, if the first group wins, the probability of introducing a new product is 0.7 and when the second group wins, the corresponding probability is 0.3. Find the probability that the new product introduced was by the second group.

7. A bag A contains 1 white and 6 red balls. Another bag contains 4 white and 3 red balls. One of the bags is selected at random and a ball is drawn from it, which is found to be white. Find the probability that the ball drawn is from the bag A. **[CBSE 2005]**

8. There are two bags I and II. Bag I contains 3 white and 4 black balls, and bag II contains 5 white and 6 black balls. One ball is drawn at random from one of the bags and is found to be white. Find the probability that it was drawn from bag I. **[CBSE 2005C]**

9. A box contains 2 gold and 3 silver coins. Another box contains 3 gold and 3 silver coins. A box is chosen at random, and a coin is drawn from it. If the selected coin is a gold coin, find the probability that it was drawn from the second box. **[CBSE 2004C]**

10. Three urns A, B and C contain 6 red and 4 white; 2 red and 6 white; and 1 red and 5 white balls respectively. An urn is chosen at random and a ball is drawn. If the ball drawn is found to be red, find the probability that the ball was drawn from the urn A. **[CBSE 2004]**

11. Three urns contain 2 white and 3 black balls; 3 white and 2 black balls, and 4 white and 1 black ball respectively. One ball is drawn from an urn chosen at random and it was found to be white. Find the probability that it was drawn from the first urn.

12. There are three boxes, the first one containing 1 white, 2 red and 3 black balls; the second one containing 2 white, 3 red and 1 black ball and the third one containing 3 white, 1 red and 2 black balls. A box is chosen at random and from it two balls are drawn at random. One ball is red and the other, white. What is the probability that they come from the second box?

13. Urn A contains 7 white and 3 black balls; urn B contains 4 white and 6 black balls; urn C contains 2 white and 8 black balls. One of these urns is chosen at random with probabilities 0.2, 0.6 and 0.2 respectively. From the chosen urn, two balls are drawn at random without replacement. Both the balls happen to be white. Find the probability that the balls drawn are from the urn C.

14. There are 3 bags, each containing 5 white and 3 black balls. Also, there are 2 bags, each containing 2 white and 4 black balls. A white ball is drawn at random. Find the probability that this ball is from a bag of the first group.

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15. There are four boxes, A, B, C and D, containing marbles. A contains 1 red, 6 white and 3 black marbles; B contains 6 red, 2 white and 2 black marbles; C contains 8 red, 1 white and 1 black marbles; and D contains 6 white and 4 black marbles. One of the boxes is selected at random and a single marble is drawn from it. If the marble is red, what is the probability that it was drawn from the box A?

16. A car manufacturing factory has two plants X and Y. Plant X manufactures 70% of the cars and plant Y manufactures 30%. At plant X, 80% of the cars are rated of standard quality and at plant Y, 90% are rated of standard quality. A car is picked up at random and is found to be of standard quality. Find the probability that it has come from plant X. **[CBSE 2005, '06C]**

17. An insurance company insured 2000 scooters and 3000 motorcycles. The probability of an accident involving a scooter is 0.01 and that of a motorcycle is 0.02. An insured vehicle met with an accident. Find the probability that the accidented vehicle was a motorcycle. **[CBSE 2005]**

18. In a bulb factory, machines A, B and C manufacture 60%, 30% and 10% bulbs respectively. Out of these bulbs 1%, 2% and 3% of the bulbs produced respectively by A, B and C are found to be defective. A bulb is picked up at random from the total production and found to be defective. Find the probability that this bulb was produced by the machine A. **[CBSE 2008]**

ANSWERS (EXERCISE 30)

1. $\frac{2}{25}$ **2.** $\frac{68}{81}$ **3.** $\frac{3}{11}$ **4.** $\frac{3}{7}$ **5.** $\frac{2}{3}$ **6.** $\frac{2}{9}$ **7.** $\frac{1}{5}$ **8.** $\frac{33}{68}$

9. $\frac{5}{9}$ **10.** $\frac{36}{61}$ **11.** $\frac{2}{9}$ **12.** $\frac{6}{11}$ **13.** $\frac{1}{40}$ **14.** $\frac{45}{61}$ **15.** $\frac{1}{15}$ **16.** $\frac{56}{83}$

17. $\frac{3}{4}$ **18.** $\frac{2}{5}$

HINTS TO SOME SELECTED QUESTIONS (EXERCISE 30)

2. Let E_1 = event that the selected scooter is produced at plant A, and
E_2 = event that the selected scooter is produced at plant B.

Then, $P(E_1) = \frac{80}{100} = \frac{4}{5}$ and $P(E_2) = \frac{20}{100} = \frac{1}{5}$.

Let E be the event of choosing a scooter which is of standard quality.

Then, $P(E/E_1) = \frac{85}{100} = \frac{17}{20}$, and $P(E/E_2) = \frac{65}{100} = \frac{13}{20}$.

Probability that the selected scooter was produced at plant A, given that it is of standard quality

$$= P(E_1/E)$$

$$= \frac{P(E/E_1)\cdot P(E_1)}{P(E/E_1)\cdot P(E_1) + P(E/E_2)\cdot P(E_2)}.$$

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3. Let E_1 and E_2 be the events of selecting a boy and a girl respectively.

Then, $P(E_1) = \frac{40}{100} = \frac{2}{5}$, and $P(E_2) = \frac{60}{100} = \frac{3}{5}$.

Let E = event that the student selected is taller than 1.75 m.

Then, $P(E/E_1) = \frac{4}{100} = \frac{1}{25}$ and $P(E/E_2) = \frac{1}{100}$.

Probability that the selected student is a girl, given that she is taller than 1.75 m

$$= P(E_2/E)$$
$$= \frac{P(E/E_2)\cdot P(E_2)}{P(E/E_1)\cdot P(E_1) + P(E/E_2)\cdot P(E_2)}.$$

4. Let E_1 and E_2 be the events of selecting a boy and a girl respectively. Then,

$$P(E_1) = \frac{60}{100} = \frac{3}{5}, \text{ and } P(E_2) = \frac{40}{100} = \frac{2}{5}.$$

Let E be the event of selecting a student having an IQ of more than 150.

Then, $P(E/E_1) = \frac{5}{100} = \frac{1}{20}$, and $P(E/E_2) = \frac{10}{100} = \frac{1}{10}$.

Required probability $= P(E_1/E)$

$$= \frac{P(E/E_1)\cdot P(E_1)}{P(E/E_1)\cdot P(E_1) + P(E/E_2)\cdot P(E_2)}.$$

5. Let there be 1000 males and 1000 females.

Let E_1 and E_2 be the events of choosing a male and a female respectively. Then,

$$P(E_1) = \frac{1000}{2000} = \frac{1}{2}, \text{ and } P(E_2) = \frac{1000}{2000} = \frac{1}{2}.$$

Let E be the event of choosing a grey haired person. Then,

$$P(E/E_1) = \frac{50}{1000} = \frac{1}{20}, \text{ and } P(E/E_2) = \frac{25}{1000} = \frac{1}{40}.$$

Probability of selecting a male person, given that the person selected is a grey haired

$$= P(E_1/E)$$
$$= \frac{P(E/E_1)\cdot P(E_1)}{P(E/E_1)\cdot P(E_1) + P(E/E_2)\cdot P(E_2)}.$$

6. Let E_1 = event that the first group wins,

E_2 = event that the second group wins, and

E = event that a new product is introduced.

Then, $P(E_1) = 0.6$, $P(E_2) = 0.4$,

$P(E/E_1) = 0.7$, $P(E/E_2) = 0.3$.

Required probability $= P(E_2/E)$

$$= \frac{P(E/E_2)\cdot P(E_2)}{P(E/E_1)\cdot P(E_1) + P(E/E_2)\cdot P(E_2)}.$$

10. Let E_1, E_2 and E_3 be the events of choosing the urns A, B and C respectively, and let E be the event of drawing a red ball. Then,

$$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}.$$

$$P\left(\frac{E}{E_1}\right) = \frac{6}{10}, \quad P\left(\frac{E}{E_2}\right) = \frac{2}{8}, \text{ and } \quad P\left(\frac{E}{E_3}\right) = \frac{1}{6}.$$

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Required probability $= P\left(\frac{E_1}{E}\right)$

$$= \frac{P(E_1)\times P\left(\frac{E}{E_1}\right)}{P(E_1)\times P\left(\frac{E}{E_1}\right)+P(E_2)\times P\left(\frac{E}{E_2}\right)+P(E_3)\times P\left(\frac{E}{E_3}\right)}$$

$$= \frac{\left(\frac{1}{3}\times\frac{6}{10}\right)}{\left(\frac{1}{3}\times\frac{6}{10}\right)+\left(\frac{1}{3}\times\frac{2}{8}\right)+\left(\frac{1}{3}\times\frac{1}{6}\right)} = \frac{36}{61}.$$

13. $P(A)=0.2,\ P(B)=0.6$ and $P(C)=0.2$.

Let E be the event that 2 white balls are drawn. Then,

$$P(E/A)=\frac{{}^7C_2}{{}^{10}C_2};\ P(E/B)=\frac{{}^4C_2}{{}^{10}C_2};\ P(E/C)=\frac{{}^2C_2}{{}^{10}C_2}.$$

$\therefore$ required probability

$$= P(C/E)=\frac{P(E/C)\cdot P(C)}{P(E/A)\cdot P(A)+P(E/B)\cdot P(B)+P(E/C)\cdot P(C)}.$$

14. Let E_1 = event of selecting a bag from the first group, and
E_2 = event of selecting a bag from the second group.

Then, $P(E_1)=\frac{3}{5}$ and $P(E_2)=\frac{2}{5}$.

Let E = event that the ball drawn is white. Then,

$$P(E/E_1)=\frac{5}{8},\ P(E/E_2)=\frac{2}{6}=\frac{1}{3}.$$

$$\therefore\quad P(E_1/E)=\frac{P(E/E_1)\cdot P(E_1)}{P(E/E_1)\cdot P(E_1)+P(E/E_2)\cdot P(E_2)}.$$

15. Let E_1, E_2, E_3, E_4 be the events of selecting boxes A, B, C, D respectively. Then,

$$P(E_1)=P(E_2)=P(E_3)=P(E_4)=\frac{1}{4}.$$

Let E = event that the marble drawn is red. Then,

$$P(E/E_1)=\frac{1}{10},\ P(E/E_2)=\frac{6}{10}=\frac{3}{5},\ P(E/E_3)=\frac{8}{10}=\frac{4}{5},\ P(E/E_4)=0.$$

$$\therefore\quad P(E_1/E)=\frac{P(E/E_1)\cdot P(E_1)}{P(E/E_1)\cdot P(E_1)+P(E/E_2)\cdot P(E_2)+P(E/E_3)\cdot P(E_3)+P(E/E_4)\cdot P(E_4)}$$

16. Let E_1 and E_2 be the events that the car is manufactured by plant X and Y respectively. Let E be the event that the car is of standard quality. Then,

$$P(E_1)=\frac{70}{100}=\frac{7}{10},\ P(E_2)=\frac{30}{100}=\frac{3}{10};$$

$$P(E/E_1)=\frac{80}{100}=\frac{4}{5},\ P(E/E_2)=\frac{90}{100}=\frac{9}{10}.$$

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$$\therefore \quad P(E_1/E) = \frac{P(E_1) \times P(E/E_1)}{P(E_1) \times P(E/E_1) + P(E_2) \times P(E/E_2)}$$

$$= \frac{\left(\frac{7}{10} \times \frac{4}{5}\right)}{\left(\frac{7}{10} \times \frac{4}{5}\right) + \left(\frac{3}{10} \times \frac{9}{10}\right)} = \frac{56}{83}.$$

17. Let E_1 and E_2 be the events that an insured vehicle is a scooter and a motorcycle respectively.

Let E be the event that the insured vehicle meets an accident.

$$P(E_1) = \frac{2000}{(2000 + 3000)} = \frac{2}{5}, \; P(E_2) = \frac{3000}{5000} = \frac{3}{5},$$

$P(E/E_1) = 0.01$ and $P(E/E_2) = 0.02$.

$$\therefore \quad P(E_2/E) = \frac{P(E_2) \cdot P(E/E_2)}{P(E_1) \cdot P(E/E_1) + P(E_2) \cdot P(E/E_2)}$$

$$= \frac{\left(\frac{3}{5} \times 0.02\right)}{\left(\frac{2}{5} \times 0.01\right) + \left(\frac{3}{5} \times 0.02\right)} = \frac{3}{4}.$$

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